MAT1505-05/08 OIS QUIZ 6   Print Name (Last, First) ______________ /10

Show all work on this sheet, including indications of mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation/syntax. Label parts, $\boxed{\text{box}}$ final short answers.

11.2.52 a). A certain ball has the property that each time it falls from a height h onto a hard level surface, it rebounds to a height rh, where $0<r<1$. Suppose the ball is dropped from an initial height of H meters. Assuming the ball continues to bounce indefinitely, find the total distance that it travels.

a) Make a diagram showing the motion steps (down, up/down, ...) through at least the first two bounces, labeling each height.

b) Write down the corresponding terms of the infinite series at least through the 3rd bounce, term by term.

c) Finish the problem.

11.2.53 What is the value of c if $\sum_{n=2}^{\infty}(1+c)^{-n} = 2$?

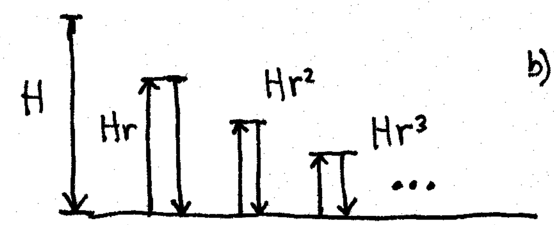
a) Write out the terms in this series at least through the first 4 terms.

b) What is the first term?

c) What is the ratio of this geometric series?

d) Finish the problem. Be sure to check that all values of c you find are actually valid solutions of this equation.

① a)



b) $d = H + \underbrace{2Hr + 2Hr^2 + 2Hr^3 + \cdots}_{\text{geometric series}}$

geometric series
$a = 2Hr$
ratio r
sum $\frac{a}{1-r}$

c) $\boxed{d = H + \frac{2Hr}{1-r}} = H\left(1+\frac{2r}{1-r}\right) = H\left(\frac{1-r+2r}{1-r}\right) = \boxed{\left(\frac{1+r}{1-r}\right)H = d}$

simplified form

② a) $\underbrace{\sum_{n=2}^{\infty}(1+c)^{-n}}$ $= 2$

$= (1+c)^{-2} + (1+c)^{-3} + (1+c)^{-4} + (1+c)^{-5} + \cdots$

b) $\boxed{a = (1+c)^{-2}}$

c) $r = \frac{(1+c)^{-3}}{(1+c)^{-2}} = (1+c)^{-3+2} = \boxed{(1+c)^{-1}}$ or notice $\sum_{n=2}^{\infty}(1+c)^{-n} = \sum_{n=2}^{\infty}\left(\frac{1}{1+c}\right)^n$

clear that $\frac{1}{1+c}$ is ratio.

d) if $|r|<1$: $2 = \frac{a}{1-r} = \frac{(1+c)^{-2}}{1-(1+c)^{-1}} \cdot \underbrace{\frac{(1+c)^2}{(1+c)^2}}_{\text{clear fractions}} = \frac{1}{(1+c)^2-(1+c)} = \frac{1}{c^2+2c+1-1-c} = \frac{1}{c^2+c}$

take reciprocals: $\frac{1}{2} = c^2+c \rightarrow c^2+c-\frac{1}{2}=0 \rightarrow 2c^2+2c-1=0$

$c = \frac{-2 \pm \sqrt{4+4\cdot 2}}{4} = \frac{-2\pm 2\sqrt{3}}{4} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}$ but must check $|r|<1$:

$r = \frac{1}{1+\left(-\frac{1}{2}\pm\frac{\sqrt{3}}{2}\right)} = \frac{1}{\frac{1}{2}\pm\frac{\sqrt{3}}{2}} = \frac{1}{\frac{1\pm\sqrt{3}}{2}} = \frac{2}{1\pm\sqrt{3}}$ $+$: $\approx .73$ ✓ $\rightarrow$ $\boxed{c = -\frac{1}{2}+\frac{\sqrt{3}}{2} = \frac{\sqrt{3}-1}{2}}$

$-$: ≈ -2.73 ⊗