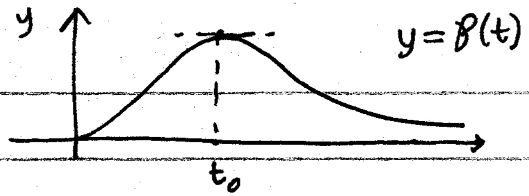
MAT 1505 015 inclass exercise

$$\rho(t) = Ate^{-ct}$$



a) Evaluate $\int Ate^{-ct}\,dt$

b) Show that $A=c^2$ makes $\int_0^\infty \rho(t)\,dt = 1$.

c) Use calculus to find the time t_0 at which the probability density function peaks.

d) What is the probability $P(0 \le t \le t_0)$, ie, evaluate $\int_0^{t_0} \rho(t)\,dt$ for $\rho(t) = c^2te^{-ct}$. Give the exact value and numerical approximation (floating point value to 3 decimal places) to this question.

e) Repeat for $P(0 \le t \le 2t_0)$.

f) The value of t for which this cumulative probability is ½ is called the median: $P(0 \le t \le t_M) = ½$. From parts d) and e), does the median satisfy $0 \le t_M \le t_0$ or $t_0 \le t_M \le 2t_0$ or $2t_0 \le t_M$?

g) $$\frac{1}{2} = \int_0^{t_M} c^2te^{-ct}\,dt$$

Let $u = ct$ and $U = ct_M$. Re-express this integral ~~expression~~ in terms of u and U. Notice that the result is equivalent to setting $c=1$.

h) For $c=1$, plot ½ and the function of t_M which results from evaluating $\int_0^{t_M} te^{-t}\,dt$ and determine the value of t_M at which the two curves intersect. [This is a check on f) since it tells you in units of t_0 what the median is.]