MAT 1505-05/08 01S QUIZ 1 Print Name (Last, First) ________________ /10

Show all work on this sheet, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation/syntax. Label parts, $\boxed{\text{box}}$ final short answers.

Consider $\int_{-1}^{3} 2x-1\,dx$: a) Draw a labeled diagram shading the region whose "net area" is represented by this integral.

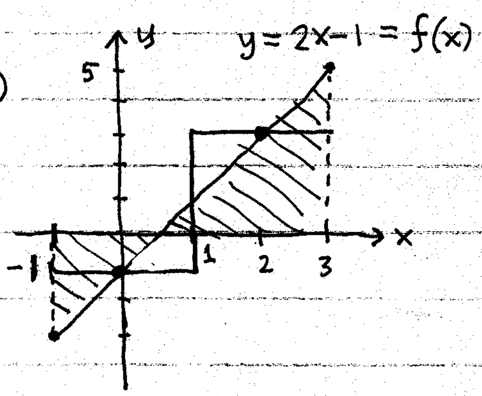
b) Now include the $n=2$ division midpoint evaluation retangles in your diagram and evaluate the corresponding Riemann sum approximation to the integral.

c) Evaluate the integral using "rules of integration."

d) Evaluate the integral using the limit Riemann sum definition (right endpoints).

[Recall $\sum_{i=1}^{n} 1 = n$, $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$, $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.]

a) b)



b) $R_{2,\text{mid}} = 2f(0) + 2f(2)$

$= 2(-1) + 2(3) = 6-2 = \boxed{4}$

c) $\int_{-1}^{3} (2x-1)\,dx = 2\left(\frac{x^2}{2}\right) - x \Big|_{-1}^{3} = (x^2-x)\Big|_{-1}^{3}$

$= (9-3) - (\underbrace{(-1)^2}_{1} \underbrace{-(-1)}_{+1}) = 6-2 = \boxed{4}$

d) $\Delta x = \frac{3-(-1)}{n} = \frac{4}{n}$

Note: For a linear function, the midpoint approximation turns out to be exact.

$x_i = -1 + i\Delta x = -1 + \frac{4i}{n}$

$f(x_i) = 2(x_i) - 1 = 2\left(-1+\frac{4i}{n}\right) - 1$

$= -2 + \frac{8i}{n} - 1 = -3 + \frac{8i}{n}$

$\sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n} \left(-3+\frac{8i}{n}\right)\frac{4}{n} = \frac{4}{n}\sum_{i=1}^{n}\left(-3+\frac{8i}{n}\right) = \frac{4}{n}\left(\sum_{i=1}^{n} -3 + \sum_{i=1}^{n}\frac{8i}{n}\right)$

$= \frac{4}{n}\left(-3\sum_{i=1}^{n} 1 + \frac{8}{n}\sum_{i=1}^{n} i\right) = \frac{4}{n}\left(-3(n) + \frac{\cancel{8}^{4}}{\cancel{n}}\left(\frac{\cancel{n}(n+1)}{\cancel{2}}\right)\right)$

$= \frac{4}{n}\left(-3n + 4(n+1)\right) = 4\left(-\frac{3n}{n} + 4\left(\frac{n+1}{n}\right)\right) = 4\left(-3 + 4\left(1+\frac{1}{n}\right)\right)$

$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = 4(-3 + 4(1+0)) = 4(4-3) = \boxed{4}$