MAT1505-05/08 01S TEST 1B Answers

① a) $\int 12t\sin 6t\,dt = 12\int \underbrace{t}_{u}\underbrace{\sin 6t\,dt}_{dv} = 12\left[\underbrace{t}_{u}\underbrace{\left(-\tfrac{1}{6}\cos 6t\right)}_{v} - \int \underbrace{\left(-\tfrac{1}{6}\cos 6t\right)}_{v}\underbrace{dt}_{du}\right]$

$u=t \quad dv=\sin 6t\,dt$

$\frac{du}{dt}=1 \quad v=\int \sin 6t\,dt$

$du=dt \quad = -\frac{1}{6}\cos 6t$

$= -2t\cos 6t + 2\int \cos 6t = -2t\cos 6t + 2\left(\tfrac{1}{6}\sin 6t\right)+C$

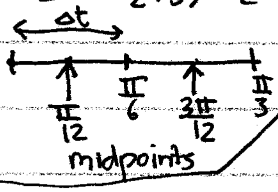
$= \boxed{-2t\cos 6t + \tfrac{1}{3}\sin 6t + C}$

b) $\int_0^{\pi/3} 12t\sin 6t\,dt = -2t\cos 6t + \tfrac{1}{3}\sin 6t\Big|_0^{\pi/3} = -\frac{2\pi}{3}\underbrace{\cos 2\pi}_{1} + \frac{1}{3}\underbrace{\sin 2\pi}_{0} - \left(0+\tfrac{1}{3}(0)\right) = \boxed{-\frac{2\pi}{3}}$ $\boxed{\approx 2.094}$

c) $f_{avg} = \frac{1}{(\frac{\pi}{3}-0)}\int_0^{\pi/3} f(t)\,dt = \frac{-2\pi/3}{\pi/3} = \boxed{-2}$

d) $M_2 = \frac{1}{2}\left(\frac{\pi}{3}\right)\cdot\left[f\left(\frac{\pi}{12}\right)+f\left(\frac{3\pi}{12}\right)\right] = \frac{\pi}{6}\left[\pi-3\pi\right] = \frac{\pi}{6}(-2\pi) = -\frac{\pi^2}{3}$, $f_{avg} \approx \frac{M_2}{(\pi/3-0)} = \frac{-\pi^2/3}{\pi/3}$

$= \boxed{-\pi \approx -3.14}$



② a) $\int \frac{x}{(8-2x)^{1/3}}\,dx = \int \underbrace{x}_{(8-u)/2}\underbrace{(8-2x)}_{u}^{-1/3}\underbrace{dx}_{-\frac{1}{2}du} = \int \frac{8-u}{2}u^{-1/3}\left(-\tfrac{1}{2}du\right)$

$u=8-2x \longrightarrow 2x=8-u \to x=\frac{8-u}{2}$

$\frac{du}{dx}=-2 \quad du=-2dx$

$dx=-\frac{1}{2}du$

$= -\frac{1}{4}\int (8-u)u^{-1/3}du$

$= -\frac{1}{4}\int 8u^{-1/3}-u^{2/3}\,du = -\frac{1}{4}\left(8\frac{u^{2/3}}{2/3}-\frac{u^{5/3}}{5/3}\right)+C = \boxed{-3(8-2x)^{2/3}+\frac{3}{20}(8-2x)^{5/3}+C}$

b) $\int_0^{7/2}\frac{x}{(8-2x)^{1/3}}dx = -3(8-2x)^{2/3}+\frac{3}{20}(8-2x)^{5/3}\Big|_0^{7/2} = -3(1)+\frac{3}{20}(1) - \left(-3\cdot 4+\frac{3}{20_5}\cdot 32^{8}\right)$

$\left(\begin{array}{l}8-2x=0\\ \to x=4>7/2\end{array}\right)$ $8-2(\tfrac{7}{2})=1$, $8^{2/3}=2^2=4$

$= -3+12+\frac{3-96}{20} = 9-\frac{93}{20} = \boxed{\frac{87}{20} = 4.35}$

③ a) $\int \frac{x\,dx}{\underbrace{x^2-9}_{u}}$ $= \frac{du}{2}$ $= \int \frac{1}{u}\frac{du}{2} = \frac{1}{2}\int\frac{1}{u}du = \frac{1}{2}\ln|u|+C = \boxed{\frac{1}{2}\ln|x^2-9|+C}$

$u=x^2-9$

$\frac{du}{dx}=2x$

$du=2x\,dx$

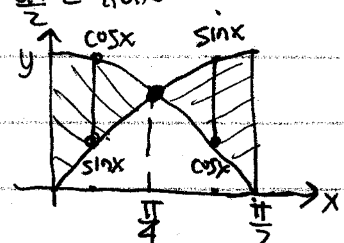
$\frac{du}{2}=x\,dx$

b) $\int_{-1}^{2}\frac{x}{x^2-9}dx = \frac{1}{2}\ln|x^2-9|\Big|_{-1}^{2} = \frac{1}{2}(\ln 5-\ln 8) = \frac{1}{2}\ln\left(\frac{5}{8}\right)$

$\left(\begin{array}{l}x^2-9=0 \to\\ x=\pm 3 \text{ outside integration interval}\end{array}\right)$

$= \ln\left(\frac{5}{8}\right)^{1/2} \neq \ln\left(\frac{5}{9}\right)^{1/2}$ $\boxed{\text{no.}}$

④



$\cos\frac{\pi}{4}=\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}$

$\text{Area} = \int_0^{\pi/4}\cos x-\sin x\,dx + \int_{\pi/4}^{\pi/2}\sin x-\cos x\,dx$

$= \sin x+\cos x\Big|_0^{\pi/4} - (\sin x+\cos x)\Big|_{\pi/4}^{\pi/2}$

$= \underbrace{\sin\frac{\pi}{4}}_{\frac{1}{\sqrt2}}+\underbrace{\cos\frac{\pi}{4}}_{\frac{1}{\sqrt2}} - (\underbrace{\sin 0}_{0}+\underbrace{\cos 0}_{1}) - (\underbrace{\sin\frac{\pi}{2}}_{1}+\underbrace{\cos\frac{\pi}{2}}_{0}) + (\underbrace{\sin\frac{\pi}{4}}_{1/\sqrt2}+\underbrace{\cos\frac{\pi}{4}}_{1/\sqrt2})$

$= 2\left(\frac{1}{\sqrt2}+\frac{1}{\sqrt2}-1\right) = 2\left(\frac{2}{\sqrt2}-1\right)$

$= \boxed{2(\sqrt2-1) \approx 1.83}$

⑤ $f(x)=3-x$, $\Delta x = \frac{3-1}{n}=\frac{2}{n}$, $x_i = 1+i\Delta x$
$=1+2i/n$

$f(x_i) = 3-x_i = 3-(1+2i/n) = 2-2i/n$

$f(x_i)\Delta x = \left(2-\frac{2i}{n}\right)\frac{2}{n}$, $\sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n}\left(2-\frac{2i}{n}\right)\frac{2}{n} = \frac{2}{n}\left(\sum_{i=1}^{n}2-\sum_{i=1}^{n}\frac{2i}{n}\right)$

$= \frac{2}{n}\left(2\left(\sum_{i=1}^{n}1\right)-\frac{2}{n}\left(\sum_{i=1}^{n}i\right)\right) = \frac{2}{n}\left(2n-\frac{2}{n}\frac{n(n+1)}{2}\right) = 2\left(2-\frac{(n+1)}{n}\right) = 2(2-1-1/n)$

$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty} 2\left(1-\frac{1}{n}\right) = \boxed{2 = \int_1^3 (3-x)\,dx}$