MAT 2705 005 Quiz 5 Print Name (Last, First) ______________ /10

Show absolutely all work (no scratch paper calculations omitted or mental calculations unreported) on this sheet in a clearly organized way, labeling problems, parts and expressions.

① $A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 10 & 2 \\ 5 & 1 \end{bmatrix}$

a) Evaluate $\det(A)$, $\det(B)$.

b) Based on part a), which of these two matrices have an inverse and why?

c) Let C be the invertible matrix of these two. Evaluate C^{-1} using row reduction techniques.

d) Use your result to solve: $C\vec{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

② $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Use the properties of the determinant under elementary row operations to derive the formula for $\det(A)$, justifying each step with a reason.

① a) $\det\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} = 3(2) - 1(5) = 1$ $\det\begin{bmatrix} 10 & 2 \\ 5 & 1 \end{bmatrix} = 10(1) - 2(5) = 0$

b) A has an inverse since $\det(A) \neq 0$ but B does not since $\det(B) = 0$.

c) $$[A, I_2] = \left[\begin{array}{cc|cc} 3 & 1 & 1 & 0 \\ 5 & 2 & 0 & 1 \end{array}\right] \to \begin{bmatrix} 1 & \frac{1}{3} & \frac{1}{3} & 0 \\ 5 & 2 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & -\frac{5}{3} & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 2 & -1 \\ 0 & \frac{1}{3} & -\frac{5}{3} & 1 \end{bmatrix}$$

$-5 \;\; -\frac{5}{3} \;\; -\frac{5}{3} \;\; 0$

$$\to \begin{bmatrix} \underbrace{\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}}_{I_2} & \underbrace{\begin{matrix} 2 & -1 \\ -5 & 3 \end{matrix}}_{A^{-1}} \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$$

d) $$A\vec{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \longrightarrow A^{-1}A\vec{x} = A^{-1}\begin{bmatrix} 1 \\ 2 \end{bmatrix} \to \vec{x} = A^{-1}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2-2 \\ -5+6 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$\vec{x} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ is the soln.

② $$\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \det\begin{bmatrix} a\cdot 1 & a\cdot\frac{b}{a} \\ c & d \end{bmatrix} = a\det\begin{bmatrix} 1 & \frac{b}{a} \\ c & d \end{bmatrix} = a\det\begin{bmatrix} 1 & b/a \\ 0 & d-\frac{bc}{a} \end{bmatrix}$$

$-c \;\; -\frac{cb}{a}$

common factor of row becomes factor of det

add row op does not change det

$$= a(1)\left(d - \frac{bc}{a}\right) = ad - bc \quad \text{(simplification)}$$

det(upper triangular matrix) = product(diag values)

we assumed $a \neq 0$ in deriving this, but in fact the formula is true independent of this assumption.