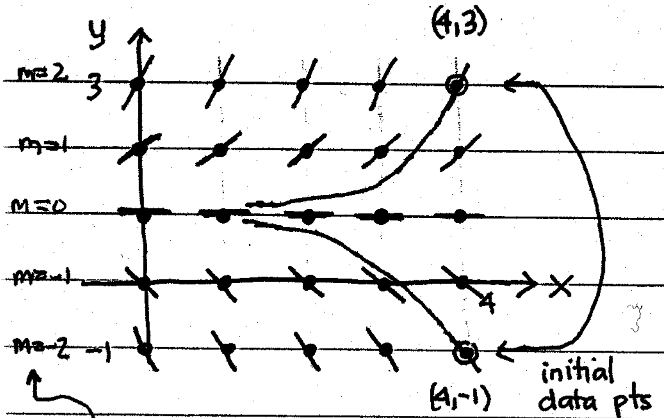
MAT2705 005 Quiz 1   Print Name (Last, First) jantzen, bob   /10



DE: $\frac{dy}{dx} = y - 1$

a) On the $0 \le x \le 4$, $-1 \le y \le 3$ unit coordinate grid points shown, draw the slope field for this DE.

b) Give a rough guess sketch of the two solution curves with initial data $y(4)=3$ and $y(4)=-1$.

c) Check that $y = 1 + ce^x$ is a solution of the D.E.

d) Find the particular solution for which $y(4)=3$.

a) slope field independent of x   b) see grid

c) $y = 1 + ce^x$ $\frac{dy}{dx} = y - 1 \rightarrow ce^x \overset{?}{=} (1+ce^x) - 1 = ce^x$ ✓

$\frac{dy}{dx} = 0 + c\frac{d}{dx}e^x = ce^x$

"best form":

d) $3 = y(4) = 1 + ce^4 \rightarrow 2 = ce^4 \rightarrow \underbrace{C = 2/e^4}_{\text{don't stop here, backsubstitute}} \longrightarrow \underbrace{y = 1 + \frac{2}{e^4}e^x}_{\text{this is what was requested}} \quad \left(= 1 + 2e^{x-4}\right)$

Remark c) "check" ≠ "derive"

It is easy to make an error deriving a solution to an equation using some procedure. The only way to check that an error was not made is by seeing whether or not your "solution" actually solves the equation it is supposed to be a solution of.

example: quadratic equation $x^2 + x - 2 = 0$

(i) derive soln, use quadratic formula incorrectly: $x = -1 \pm \sqrt{1-4(-2)} = -1 \pm \sqrt{9} = -1 \pm 3 = 2, -4$

check solns: $2^2 + 2 - 2 = 4 + 0 = 4 \ne 0$! not a soln

$(-4)^2 + (-4) - 2 = 16 - 6 = 10 \ne 0$! not a soln

(ii) derive soln, use quadratic formula correctly: $x = \frac{-1 \pm \sqrt{1-4(-2)}}{2} = \frac{-1 \pm \sqrt{9}}{2} = \frac{-1 \pm 3}{2} = 1, -2$

check solns: $1^2 + 1 - 2 = 2 - 2 = 0$ ✓

$(-2)^2 + (-2) - 2 = 4 - 4 = 0$ ✓

For a 1st order DE: $F(x, y, y') = G(x, y, y')$

to check if $y = y(x)$ is a soln, substitute everywhere: $F(x, y(x), y'(x)) = G(x, y(x), y'(x))$ and simplify until you see both sides agree. If they are not equivalent, it is not a soln. This requires first calculating the derivative $y'(x)$ as a side calculation.