MATH 2705 005 TEST 2 Print Name (Last, First) ______________________ /10

Show all work, including as many indications of mental steps as possible, in a clearly organized presentation on the other lined paper provided. Label each problem and part, clearly separating parts, $\boxed{\text{boxing}}$ any short final responses requested by each part. Try to identify expressions you write down with appropriate symbols for them linked by an equal sign.

① $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ 2 & 2 \\ 3 & 1 \end{bmatrix}$, $B^T = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}$

a) Evaluate $B^T B$.

b) Check that $A^{-1} = \begin{bmatrix} 1 & -2 & 5 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$ is the inverse of A. $\left([A^{-1}]_{23} = -4\right)$

c) If $AX = B$, what dimensions must the matrix X have and why?

d) Use the matrix A^{-1} and matrix multiplication to evaluate the matrix X of part c).

② $2x_1 + x_2 - x_4 = 2$
$x_1 + 2x_3 + x_4 = 3$
$2x_1 + x_3 + 5x_4 = 0$

a) Put this system of equations in the matrix form $A\vec{x} = \vec{b}$ and state its augmented matrix C.

b) Use row reduction techniques (to fully reduced rref form) to solve this system, if consistent, giving some indication about your individual steps. Give your final solution in the scalar form $x_1 = \cdots$, $x_2 = \cdots$, ….

③ $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ -1 & 1 & 1 & -1 \end{bmatrix}$

a) Use only elementary row operation techniques to evaluate $\det(A)$, giving some indication of what elementary row operations you use.

b) From your result for a), does $A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$ have a unique soln? Explain.

④ $B = \begin{bmatrix} 1 & 2 & 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & -4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

a) If B is the rref form of the augmented matrix of a system of linear equations $A\vec{x} = \vec{b}$ in the variables $x_1, x_2, \ldots$, write down the corresponding scalar system of equations and solve them if consistent. Give your final solution in the vector form $\vec{x} = \cdots$.

b) If B is the rref form of the coefficient matrix A of a system of linear equations $A\vec{x} = \vec{0}$ in the variables $x_1, x_2, \ldots$, write down the corresponding scalar system of equations and solve them if consistent. Give your final solution in the vector form $\vec{x} = \cdots$.

c) What is the value of rank(A) for the matrix A of part b)? Why?