MAT1500-03/11 00F Quiz9 PrintName(Last,First) ________ /10

Show all work on this sheet, including indications of mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation. $\boxed{\text{Box}}$ final short answers.

① Discuss the curve $y = x^4 - 4x^3$ with respect to:

a) its increasing/decreasing behavior, critical points, local maxima and minima,

b) concavity and points of inflection, and

c) use this information to sketch the curve, labeling all important points by both coordinates (x_0, y_0).

Make sure you give sign diagrams for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ accompanied by the appropriate icons: / \ —•— ⌣ ⌢ Ⓘ.

①a) $y = x^4 - 4x^3$

$$\frac{dy}{dx} = 4x^3 - 12x^2 = 4x^2(x-3) = 0 \rightarrow \begin{cases} x = 0 \rightarrow y = 0 \\ x = 3 \rightarrow y = 3^4 - 4\cdot 3^3 = 3^3(3-4) = -27 \end{cases}$$

critical pts: (0,0) and (3, −27)

dy/dx — — 0 — — — 0 + +

0 3

local min at x = 1 by 1stdertest: \\•/

decreasing on (−∞, 0) and (0, 3)

increasing on (3, ∞)

b) $$\frac{d^2y}{dx^2} = 12x^2 - 24x = 12x(x-2) = 0 \rightarrow \begin{cases} x = 0 \rightarrow y = 0 \\ x = 2 \rightarrow y = 2^4 - 4\cdot 2^3 = 2^4(1-2) = -2^4 = -16 \end{cases}$$

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+ + + 0 − − − 0 + +

x: 0 2

Ⓘ Ⓘ

$\frac{d^2y}{dx^2}$ switches sign at x = 0 and x = 2 leading to points of inflection: (0,0) and (2, −16)

concave down : (0, 2)

concave up : (−∞, 0) and (2, ∞)

c)