

Show all work on this sheet, including indications of mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation. **Box** final short answers.

① Discuss the curve $y = x^4 - 4x^3$ with respect to:

- a) its increasing/decreasing behavior, critical points, local maxima and minima,
- b) concavity and points of inflection, and
- c) use this information to sketch the curve, labeling all important points by both coordinates (x_0, y_0) .

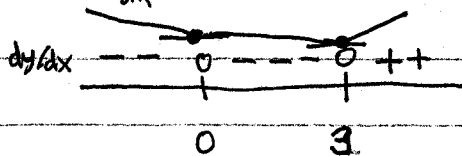
Make sure you give sign diagrams for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ accompanied by the appropriate icons: \nearrow \searrow \rightarrow \curvearrowright \curvearrowleft \textcircled{I} .

① a) $y = x^4 - 4x^3$

$\frac{dy}{dx} = 4x^3 - 12x^2 = 4x^2(x-3) = 0 \rightarrow \begin{cases} x=0 \rightarrow y=0 \\ x=3 \rightarrow y=3^4 - 4 \cdot 3^3 = 3^3(3-4) = -27 \end{cases}$

critical pts:

$(0,0)$ and $(3,-27)$

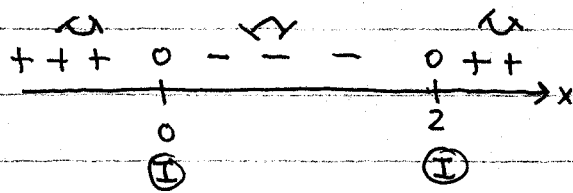


local min at $x=3$ by 1st derivative test: $\searrow \nearrow$

decreasing on $(-\infty, 0)$ and $(0, 3)$

increasing on $(3, \infty)$

b) $\frac{d^2y}{dx^2} = 12x^2 - 24x = 12x(x-2) = 0 \rightarrow \begin{cases} x=0 \rightarrow y=0 \\ x=2 \rightarrow y=2^4 - 4 \cdot 2^3 = 2^4(1-2) = -2^4 = -16 \end{cases}$



$\frac{d^2y}{dx^2}$ switches sign at $x=0$ and $x=2$ leading to points of inflection:

$(0,0)$ and $(2,-16)$

concave down : $(0,2)$

concave up : $(-\infty, 0)$ and $(2, \infty)$

