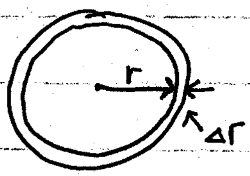
MAT 1500-03/11 00F(Q8) Print Name (Last, First) jantzen, bob /10

Show all work on this sheet, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation. $\boxed{\text{Box}}$ short final answers.

① 

a) Compute the exact area ΔA of the ring of inner radius r and outer radius $r+\Delta r$.

b) Use the differential approximation to calculate this area, assuming Δr is much smaller than r.

(c) Evaluate the error (approx−exact) in this approximation, simplifying your result.

② Find the absolute maximum of the function $f(x) = x^3 - x^2 - x$ on the interval $-10 \le x \le 1$

① a) $\left.\begin{array}{l} A(r) = \pi r^2 \\ A(r+\Delta r) = \pi(r+\Delta r)^2 \end{array}\right\}$ $\Delta A = A(r+\Delta r) - A(r) = \pi(r+\Delta r)^2 - \pi r^2$

$= \pi\left[(r+\Delta r)^2 - r^2\right] = \pi\left[\cancel{r^2} + 2r\Delta r + \Delta r^2 - \cancel{r^2}\right]$

$= \boxed{2\pi r\Delta r + \pi\Delta r^2}$

b) $A = \pi r^2$

$\frac{dA}{dr} = \frac{d}{dr}(\pi r^2) = \pi(2r) = 2\pi r$

$dA = 2\pi r \underbrace{dr}_{\Delta r} = \boxed{2\pi r\Delta r}$

c) $dA - \Delta A = 2\pi r\Delta r - (2\pi r\Delta r + \pi\Delta r^2) = 2\pi r\Delta r - 2\pi r\Delta r - \pi\Delta r^2$

$= \boxed{-\pi\Delta r^2} < 0$ so $dA < \Delta A$ (approximation is low)

② $f(x) = x^3 - x^2 - x$ on $[-10, 1]$.

$f'(x) = 3x^2 - 2x - 1 = (3x+1)(x-1) = 0$

$3x + 1 = 0$ or $x - 1 = 0$

$\underbrace{x = -1/3}$ or $x = 1$

one critical number for f in $(-10, 1)$

$f(-10) = (-10)^3 - (-10)^2 - (-10) = -1000 - 100 + 10 = -1090$

$f(-1/3) = (-\frac{1}{3})^3 - (-\frac{1}{3})^2 - (-\frac{1}{3}) = -\frac{1}{27} - \frac{1}{9} + \frac{1}{3} = \frac{-1-3+9}{27} = \frac{5}{27}$ → largest value among endpt/critical values

$f(1) = 1^3 - 1^2 - 1 = -1$

$\boxed{\text{abs max is } \frac{5}{27} \text{ occurring at } x = -1/3}$