MAT1500-03/11 Quiz 4 Print Name (Last, First) ____________ /10

Show all work on this sheet, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation. $\boxed{\text{Box}}$ final short answers.

$$f(x) = 2x^2-3, \quad g(x) = \frac{3x^2-1}{1-x^2}$$

① a) Find the slope of the tangent line to f at a (or at "$x=a$"). [using limits]

b) Use your result to write the equation of the tangent line to f at $x=1$, and give your final result with y expressed as a function of x.

② a) Evaluate the limits necessary to determine if g has any horizontal asymptotes.

b) Give the equations of any horizontal asymptotes you find.

① a) $$m = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

$$\begin{cases} f(x) = 2x^2-3 \\ f(a) = 2a^2-3 \\ f(a+h) = 2(a+h)^2-3 \\ f(a+h)-f(a) = [2(a+h)^2-3]-[2a^2-3] \end{cases}$$

$$\frac{[2(a+h)^2-3]-[2a^2-3]}{h} = \frac{2(a^2+2ah+h^2)-3-2a^2+3}{h}$$

$$= \frac{\cancel{2a^2}+4ah+2h^2\cancel{-3}\cancel{-2a^2}\cancel{+3}}{h} = \frac{4ah+2h^2}{h} = \frac{h(4a+2h)}{h} \overset{h\neq 0}{=} 4a+2h$$

so $m = \lim_{h\to 0}(4a+2h) = 4a+2(0) = \boxed{4a}$

b) Setting $a=1$: $m = 4(1) = 4$.

If $x=1$, then $y=f(1) = 2(1)^2-3 = 2-3 = -1$

pt $(1,-1)$, slope $4 \longrightarrow$ pt-slope eq: $y-(-1) = 4(x-1)$

$y+1 = 4x-4$

$\boxed{y = 4x-5}$

② a) $$\lim_{x\to\infty} \frac{3x^2-1}{1-x^2} = \lim_{x\to\infty} \frac{(3x^2-1)/x^2}{(1-x^2)/x^2} = \lim_{x\to\infty} \frac{3-\cancel{1/x^2}^{\to 0}}{\cancel{1/x^2}_{\to 0}-1} = \frac{3}{-1} = -3$$

$\lim_{x\to-\infty} \frac{3x^2-1}{1-x^2} = -3$ (same calculation, no change since g is even)

b) $\boxed{y=-3}$ is a horizontal asymptote for g both as $x\to\infty$ and $x\to-\infty$.