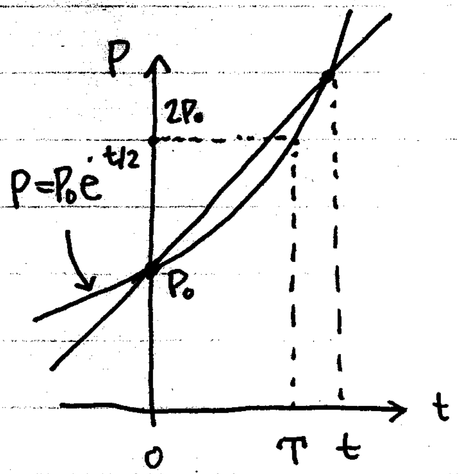
MAT 1500-03/11 00F QUIZ 3 Print Name (Last, First) Jantzen, bob /10

Show all work on this sheet, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation/syntax. $\boxed{\text{Box}}$ short final answers.



① a) For the graph of population versus time shown, find an expression for the slope of the secant line shown in the figure (ie., between $t=0$ and a general value of t).

b) Write an expression involving limit notation for the slope m_{tan} of the tangent line at $t=0$.

c) Later we will discover that $m_{tan} = P_0/2$. Given this fact, write the equation for the tangent line at $t=0$ (in the form $P =$ "linear function"(t).)

d) Find a formula for the time T it takes for the population to double (compared to its value at $t=0$).

① a) $m_{sec} = \dfrac{\text{change}(P)}{\text{change}(t)} = \dfrac{P_0 e^{t/2} - P_0 e^{0/2}}{t-0} = \boxed{\dfrac{P_0 e^{t/2} - P_0}{t} = \dfrac{P_0(e^{t/2}-1)}{t}}$ either acceptable

b) $m_{tan} = \lim\limits_{t\to 0} m_{sec} = \boxed{\lim\limits_{t\to 0} \dfrac{P_0 e^{t/2} - P_0}{t} = \lim\limits_{t\to 0} \dfrac{P_0(e^{t/2}-1)}{t}}$ either acceptable

c) point $(0, P_0)$ or vertical intercept P_0 plus slope $P_0/2$:

$\boxed{P = \underbrace{\tfrac{P_0}{2}}_{\text{slope}} t + \underbrace{P_0}_{\text{intercept}}}$ (slope intercept form) $= \boxed{P_0\left(1+\tfrac{t}{2}\right)}$ equivalent form

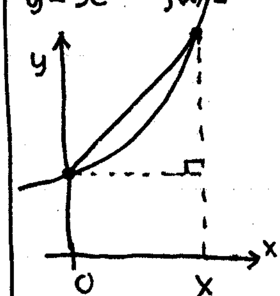
d) $\underbrace{2P_0}_{\substack{\text{doubled}\\ \text{population}}} = \underbrace{P_0 e^{T/2}}_{\text{at time } T} \rightarrow 2 = e^{T/2} \rightarrow \ln 2 = \ln e^{T/2} = T/2$

(since at $t=0$, $P=P_0$)

$\boxed{T = 2\ln 2}$

Compare:

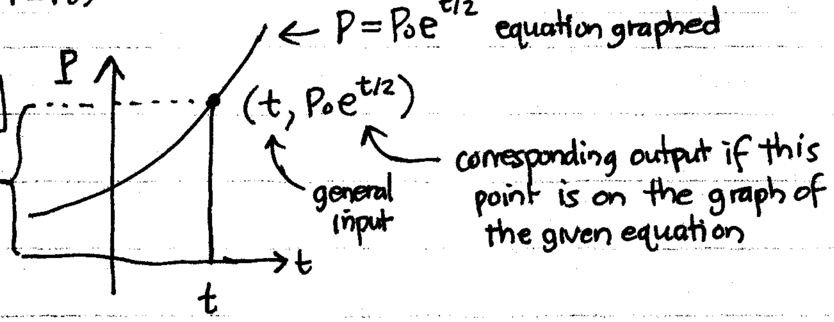
$y = 3e^{x/2} = f(x)$



$m_{sec} = \dfrac{y|_x - y|_{x=0}}{x-0} = \dfrac{f(x)-f(0)}{x-0} = \dfrac{3e^{x/2} - 3e^{0/2}}{x-0} = \dfrac{3e^{x/2}-3}{x}$

$m_{tan} = \lim\limits_{x\to 0} \dfrac{3e^{x/2}-3}{x}$

Notes:



Note that there are two functional relationships in this problem $P = P_0 e^{t/2}$ and $P = \frac{P_0}{2}t + P_0$, so cannot assume that P or $P(t)$ is well-defined in a formula.