MAT1500-03/11 00F QUIZ 2 Print Name (Last, First): jantzen, bob /10

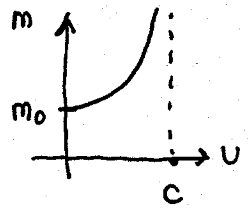
Show all work on this sheet, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical syntax/notation. $\boxed{\text{Box}}$ short final answers.

① Solve for x : $e^{3x-4} = 2$.

② The relativistic mass $m > 0$ of a particle with speed $v \geq 0$ is:

$$m = f(v) = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}},$$

where $m_0 > 0$ is the rest mass of the particle and $c > 0$ is the speed of light in vacuum (ie, m_0 and c are just positive constants).



Find the inverse function f^{-1} (ie $v = f^{-1}(m)$) and its domain.

---

① $\ln\left[e^{3x-4} = 2\right]$

"multiply by 3, subtract 4, exponentiate"

divide by 3 ← add 4 ← ln

$$\ln e^{3x-4} = \ln 2$$

$$3x - 4 = \ln 2$$

$$3x = 4 + \ln 2$$

$$\boxed{x = \frac{4+\ln 2}{3}}$$

② $m = \dfrac{m_0}{\left(1-\frac{v^2}{c^2}\right)^{1/2}}$

cross-multiply:

$$\left(1-\frac{v^2}{c^2}\right)^{1/2} = \frac{m_0}{m}$$

square:

$$1-\frac{v^2}{c^2} = \left(\frac{m_0}{m}\right)^2 = \frac{m_0^2}{m^2}$$

add/sub:

$$1 - \frac{m_0^2}{m^2} = \frac{v^2}{c^2}$$

mult:

$$c^2\left(1-\frac{m_0^2}{m^2}\right) = v^2$$

$$\underset{v \geq 0}{v} = \sqrt{v^2} = \sqrt{c^2\left(1-\frac{m_0^2}{m^2}\right)}$$

$$= \sqrt{c^2}\sqrt{1-\frac{m_0^2}{m^2}}$$

$$= c\sqrt{1-\frac{m_0^2}{m^2}}$$

$$\boxed{v = c\sqrt{1-\frac{m_0^2}{m^2}} = f^{-1}(m)}$$

domain: $1 - \frac{m_0^2}{m^2} \geq 0$

$$1 \geq \frac{m_0^2}{m^2}$$

$$m^2 \geq m_0^2$$

$\boxed{m \geq m_0}$ (since both positive)