Mat1500-03/11 00F Final Exam Print Name (Last, First) ____________________ /100

Read the instructions at the end first.

① Differentiate the following functions [$f(x)=\dots$] or functional relationships [$y=f(x)$] and simplify your results, using appropriate derivative notation [$f'(x)=\dots$ or $\frac{dy}{dx}=\dots$]:

a) $f(x)=(2-x^2)^3$

b) $y=\frac{x}{\sqrt{1-x^2}}$ (combine terms in result)

c) $g(t)=\ln\left(\frac{t-1}{t+1}\right)$ (combine terms in result)

d) $s=e^{-t}\sin 3t$ (factor result)

e) $T=A\cos(\omega t+\delta)$; A,ω,δ constants

f) repeat b) using logarithmic differentiation and simplify your result, combining terms.

g) evaluate the second derivative of a), and factor the result.

② Find the equation of the tangent line to the curve $xy^2-x^2y=6$ at $(1,-2)$, solving for y.

③ Suppose a function has the following properties: $f(4)=5$, $f'(4)=-2$, $f''(4)=2$.

a) Write down the equation of the tangent line to the graph of f in the x-y plane at $x=4$ and solve for y.

b) Use your result to approximate $f(3.9)$ and $f(4.1)$.

c) Make a diagram locating the point and a small piece of its tangent line  in the x-y plane with the appropriate slope for the point on the graph of f corresponding to this information, and then make a small correction to indicate a small piece of the actual graph taking into account the second derivative information.

d) Are your estimates in b) too small or too large? Explain.

④ The volume of a cube is increasing at a rate of 150 cm³/min. How fast is the edge length increasing when this length is 5 cm? [Respond in words, after working the problem.]

⑤ The formula for relativistic momentum p of a particle of rest mass m_o and speed v is $p=\frac{mv}{\sqrt{1-v^2/c^2}}$, where c is the speed of light. This can be rewritten $\frac{p}{mc}=\frac{v/c}{\sqrt{1-(v/c)^2}}$ or $y=\frac{x}{\sqrt{1-x^2}}$, where $x=\frac{v}{c}$ and $y=\frac{p}{mc}$ are dimensionless variables. The familiar nonrelativistic limit occurs for speeds small compared to c, i.e., $v<<c$ or $\frac{v}{c}<<1$ or $x<<1$ (x very small compared to 1).

a) Show that $y=\frac{x}{\sqrt{1-x^2}}\approx x$ using the linear approximation at $x=0$, for small enough x.

b) Evaluate $\lim_{x\to1^-}\frac{x}{\sqrt{1-x^2}}$. (Explain.)

⑥ Given $f(x)=xe^{-x}$, $f'(x)=(1-x)e^{-x}$, $f''(x)=(x-2)e^{-x}$, discuss intercepts (show work), asymptotes (evaluate carefully appropriate limits), critical points and points of inflection (give both coordinates of each such point) and local and global extrema (explain). In your discussion provide a sign line (+−0) for f' with a stick figure graph above it and a sign line (+−0) for f'' with icons above it. List intervals of increase/decrease and concavity up/down. Make a rough graph of f locating all key points and labeling everything in your diagram.

Instructions

Box each final short answer and nothing else. (Exclude graphs and explanations).
Use proper mathematical notation and clearly organize your work. Label each part.
Try to imagine another student trying to follow your work but needing as much help as possible to understand, so try to justify every step.
Reread each part after finishing it to make sure you've responded correctly.
If you have time, check your work with MAPLE after completing the test.

The only software you are allowed to open on your computer is MAPLE.
No browser. No e-mail. Read the pledge below.

When you have completed the exam, please reread and sign:

During this examination all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until the exam period is terminated.

Signature:

Date: