

Read instructions on the reverse side.

- ① Verify that $\frac{1}{\sqrt{1-x}} \approx 1 + \frac{x}{2}$ is the correct linear approximation at $x=0$.
- ② Find all value(s) of c (if any) which satisfy the conclusion of the Mean Value Theorem for the function $f(x) = \frac{1}{1+x}$ on the interval $[0,1]$.
- ③ A driver passes through a turnpike toll booth (not the E-ZPass lane) and gets a ticket punched for 2:10 pm. Down the road 74 miles, she exits the turnpike and hands in her ticket which is punched at 3:05 pm. In addition to the toll amount, she is handed a speeding ticket for driving in excess of 80 mph in a 65 mph zone. Is this speeding ticket justifiable? Explain.
- ④ The period of a pendulum is given by the formula $T = 2\pi\sqrt{\frac{L}{g}}$, where L is the length of the pendulum in feet, $g = 32 \text{ ft/s}^2$ is the acceleration due to gravity, and T is the length of the period in seconds. If the length of the pendulum is measured to be 3 ft long to within $\pm \frac{1}{8}$ in, what is the approximate percentage error in the calculated period T ? (Here a decimal number is appropriate.)
- ⑤ Evaluate the following limits, showing clearly what you are doing in each step and why:
 - a) $\lim_{x \rightarrow 0} \frac{e^{ax} - e^{bx}}{x}$
 - b) $\lim_{x \rightarrow 1} x^{\frac{1}{1-x}}$
- ⑥ $f(x) = (x^2 - 1)^3$. Discuss the behavior of this function (symmetry, intercepts, limits at $\pm\infty$) and use sign lines with icons for f' and f'' to display the derivative information and make a rough graph of f identifying all key points and labeling them with their coordinates (x_0, y_0) and indicating what kind of points they are.
- ⑦ $f(x) = \frac{x^{1/3}}{1-x}$.
 - a) Evaluate $f'(x)$ and simplify (combine terms).
 - b) Evaluate $f''(x)$ using the quotient rule and simplify (combine terms).
 - c) Set $u = f'(x)$ and use logarithmic differentiation to evaluate $\frac{du}{dx} = f''(x)$ and simplify, as a check on part b).
 - d) Use Maple to check your work so far.*
 - e) Check the domain of f , asymptotes, symmetry, intercepts.
 - f) Make a sign line for f' with some support for your sign assignment. Make a stick figure graph of f above it. Identify all critical points, local extrema, cusps, or vertical tangents.
 - g) Make a sign line for f'' with icons and some support for your sign assignments. Identify points of inflection.
 - h) State intervals of increase/decrease and concave up/down behavior for f .
 - i) Make a rough graph of f on the basis of all of the above information, labeling the key points (x_0, y_0) and asymptotes.
 - j) Compare with an appropriate viewing window plot of f alone using MAPLE* and then with a MAPLE plot of f, f', f'' together, as a check on your work in f), g), h).

Math Exam Rules

READ THESE INSTRUCTIONS CAREFULLY

This test is not about just getting "the right answer", but also presenting and communicating well the process which leads to the results requested in each part of every problem, as well as your understanding of the course content and its vocabulary. [This is good practice for learning how to communicate technical results to other people in a workplace environment.] No results here may be justified using technology -- a reasoned explanation supported by mathematical facts is always required and cannot be substituted by a technology result. However, you are encouraged to use MAPLE to check every result you derive by hand. [For a takehome exam, no collaboration is allowed but you may consult your textbook, your notes and my handouts.] Come talk to me if you get stuck on any problem.

Show **all** work and answers, including indications of mental steps, on the lined paper provided. Put your name on each sheet and clearly label continuations of problems from one sheet to another. Label and SEPARATE clearly each part of each problem and BOX each short final response requested (and nothing else). Cross out abandoned work not to be considered.

Use proper mathematical notation: "symbol" = "expression representing symbol" = ... Don't misuse equal signs, and don't write down unidentified expressions, but do link expressions which are equal with equal signs. Give EXACT ANSWERS, not decimal approximations, unless the context warrants it, but first give the exact result in any case. Always simplify results.

When you have completed the exam, please read and sign the dr bob integrity pledge:

"During this examination, all work has been my own. I give my word as a decent human being that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated."

Signature:

Date:

MAPLE 7 d) and j):

Attach a printout of f defined as a MAPLE function and its first and second derivatives evaluated using the D notation and simplified with the `normal`(or `simplify`) command, together with the two plots where the options `numpoints = 100, discant = true` (page 23 Calc labs) may help the appearance, while the `convert(..., surd)` command is necessary to show the plot for $x < 0$. [Use `color` option for multiple curves.]