MAT1500-03/11 00F TEST2 Answers

① a) $y = x^2 e^{-x}$

$\frac{dy}{dx} = \frac{d}{dx}(x^2 e^{-x}) = \frac{d}{dx}(x^2)\, e^{-x} + x^2 \frac{d}{dx} e^{-x}$

$= 2x e^{-x} + x^2 e^{-x}(-1) = (2x - x^2) e^{-x}$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[(2x-x^2) e^{-x}\right]$

$= \frac{d}{dx}(2x-x^2)\, e^{-x} + (2x-x^2)\frac{d}{dx} e^{-x}$

$= (2-2x) e^{-x} + (2x-x^2) e^{-x}(-1)$

$= (2-2x-2x+x^2) e^{-x} = \boxed{(2-4x+x^2) e^{-x}}$

$\left.\frac{d^2y}{dx^2}\right|_{x=1} = (2-4(1)+1^2) e^{-1} = \boxed{-e^{-1}}$

b) $0 = \frac{dy}{dx} = (2x-x^2) e^{-x} = (2-x) \times e^{-x}$

$\hookrightarrow \boxed{x = 2, 0}$

② $y = \frac{1-x}{1+x}$, $\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1-x}{1+x}\right)$

$= \frac{(1+x)\frac{d}{dx}(1-x) - (1-x)\frac{d}{dx}(1+x)}{(1+x)^2} = \frac{-(1+x)-(1-x)}{(1+x)^2}$

$= \frac{-1-x-1+x}{(1+x)^2} = \frac{-2}{(1+x)^2}$ $\quad \left.\frac{dy}{dx}\right|_{x=-2} = \frac{-2}{(1-2)^2} = -2$

$y|_{x=-2} = \frac{1-(-2)}{1+(-2)} = \frac{3}{-1} = -3$

$y - (-3) = -2(x-(-2)) \rightarrow y+3 = -2(x+2)$

$\boxed{y = -2(x+2) - 3 = -2x - 7}$

③ $\ln\left[y = x^{\sin x}\right]$ $\quad \ln y = \ln x^{\sin x} = \sin x \ln x$

$\frac{d}{dx}(\ln y) = \frac{d}{dx}(\sin x \ln x)$

$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}(\sin x)\ln x + \sin x \frac{d}{dx}\ln x$

$= \cos x \ln x + \sin x \cdot \frac{1}{x}$

$\frac{dy}{dx} = y\left(\cos x \ln x + \frac{\sin x}{x}\right) = \boxed{x^{\sin x}\left(\cos x \ln x + \frac{\sin x}{x}\right)}$

$\left.\frac{dy}{dx}\right|_{x=1} = \underbrace{1^{\sin 1}}_{1}\left(\cos 1 \underbrace{\ln 1}_{0} + \frac{\sin 1}{1}\right) = \boxed{\sin 1}$

④ $\frac{d}{dx}\left[x^2 + xy + y^2 = 7\right]$

$\frac{d}{dx}x^2 + \frac{d}{dx}(xy) + \frac{d}{dx}(y^2) = 0$

$2x + \underbrace{\frac{d}{dx}(x)}_{1}\, y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$2x + y + (x+2y)\frac{dy}{dx} = 0$

$\boxed{\frac{dy}{dx} = -\frac{(2x+y)}{x+2y}}$

$\left.\frac{dy}{dx}\right|_{\substack{x=1\\y=2}} = -\frac{2(1)+2}{1+2(2)} = \boxed{-\frac{4}{5}}$

⑤ $h(\theta) = (4-3\sin^2 5\theta)^{1/2}$

$h'(\theta) = \frac{d}{d\theta}(4-3(\sin 5\theta)^2)^{1/2}$

$= \frac{1}{2}(4-3\sin^2 5\theta)^{-1/2}\frac{d}{d\theta}(4-3(\sin 5\theta)^2)$

$= \frac{1}{2(4-3\sin^2 5\theta)^{1/2}}(0-3(2\sin 5\theta)\underbrace{\frac{d}{d\theta}(\sin 5\theta)}_{\cos 5\theta \cdot 5}$

$= \boxed{\frac{-15\sin 5\theta \cos 5\theta}{(4-3\sin^2 5\theta)^{1/2}}}$

⑥ $h(x) = f(f(x))$

$h'(x) = f'(f(x))\, f'(x)$

$h(-1) = f(f(-1)) = f(4) = \boxed{3}$

($f(-1) = 4$ when $x=-1$; $f(4) = 3$ when $x=4$)

$h'(-1) = f'(f(-1))\, f'(-1)$

$= f'(4)\, f'(-1) = 3(-1) = \boxed{-3}$

slope at $x=4$: $\frac{6}{2} = 3$ $\quad$ slope at $x=-1$: $\frac{-1}{1} = -1$