MATH 1500-03/11 OOF TEST1 Answers

① a) $V_{avg} = \frac{\Delta y}{\Delta t}\Big|_{t=0 \text{ to } t=1} = \frac{f(1)-f(0)}{1-0} = \frac{[40(1)-16(1)^2]-[40(0)-16(0)^2]}{1} = \boxed{24 \text{ (ft/sec)}}$

b) $V(t) = f'(t) = \lim_{h\to 0} \frac{f(t+h)-f(t)}{h} = \lim_{h\to 0} \frac{40(t+h)-16(t+h)^2-(40t-16t^2)}{h}$, where $40(t+h)-16(t+h)^2-(40t-16t^2) = \cancel{40t}+40h-\cancel{16t^2}-32th-16h^2-\cancel{40t}+\cancel{16t^2}$, so $= \lim_{h\to 0} \frac{40h-32th-16h^2}{h}$

$= \lim_{h\to 0} \frac{\cancel{h}(40-32t-16h)}{\cancel{h}} = \lim_{h\to 0} 40-32t-16h = 40-32t-16(0) = \boxed{40-32t = V(t)}$

c) $V(0) = 40-32(0) = \boxed{40}$, $V(2) = 40-32(2) = 40-64 = \boxed{-24}$ (ft/sec)

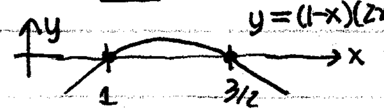
d) $0 = V(t) = 40-32t \to t = \frac{40}{32} = \frac{8\cdot 5}{8\cdot 4} = \boxed{\frac{5}{4} = 1.25 \text{ (sec)}}$ $y = f(\tfrac{5}{4}) = 40(\tfrac{5}{4}) - 16(\tfrac{5}{4})^2 = \frac{40\cdot 5}{4} - \frac{16\cdot 25}{16}$ $= \boxed{25 \text{ (ft)}}$

② $x=-1$: $y = f(-1) = (-1) - \frac{2}{(-1)} = -1+2 = 1$, $m = f'(-1) = 1 + \frac{2}{(-1)^2} = 1 + \frac{2}{1} = 3$

pt-slope: $y-1 = 3(x-(-1)) \to y = 3(x+1)+1 = 3x+3+1 = 3x+4 \to \boxed{y = 3x+4}$

③ a) input of ln must be positive: $\ln((1-x)(2x-3))$ real if $(1-x)(2x-3) = -2x^2+5x-3 > 0$

$y=(1-x)(2x-3)$ $\{=0$ at $x=1, 3/2$; graph is downward parabola, positive between zeros



$\to \boxed{1 < x < 3/2 \text{ or } (1, 3/2)}$

or: $(1-x)(2x-3) > 0$: $(1-x > 0$ and $2x-3 > 0)$ or $(1-x < 0$ and $2x-3 < 0)$

$1 > x$ and $2x > 3 \to x > 3/2$; $x < 1$ and $x > 3/2$, no soln

$1 < x$ and $2x < 3 \to x < 3/2$; $x > 1$ and $x < 3/2$

$\boxed{1 < x < \frac{3}{2}}$

b) $g(x) = \frac{6x^2+5x}{(1-x)(2x-3)}$ for vertical asymptote, denom must be zero AND numer nonzero

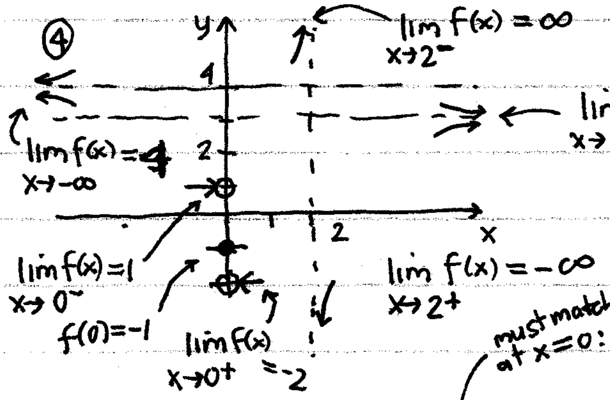
V. asymp eqs

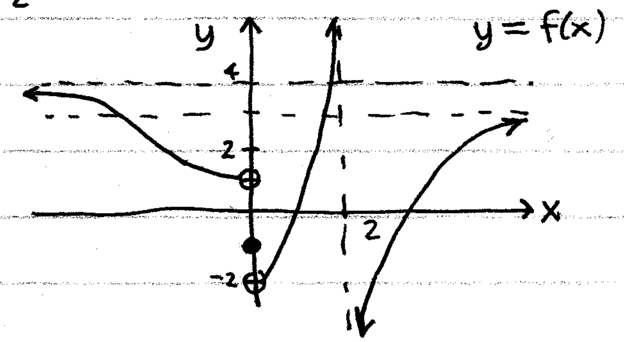
$\boxed{x=1}$: $\frac{6(1)^2+5(1)}{(-1)(2(1)-3)} = \frac{11}{0\cdot(-1)=0}$ ✓ $\lim_{x\to 1^+} \frac{6x^2+5x \approx 11}{\underbrace{(1-x)}_{\lessapprox 0}\underbrace{(2x-3)}_{\approx -1}} = +\infty$ (overall sign positive)

$\boxed{x=\frac{3}{2}}$: $\frac{6(\frac{3}{2})^2+5(\frac{3}{2}) > 0}{(1-\frac{3}{2})(2\cdot\frac{3}{2}-3) = -\frac{1}{2}\cdot 0 = 0}$ ✓ $\lim_{x\to 3/2^+} \frac{6x^2+5x > 0}{\underbrace{(1-x)}_{\approx -\frac{1}{2}}\underbrace{(2x-3)}_{\gtrapprox 0}} = -\infty$ (overall sign negative)

c) $\lim_{x\to\infty} \frac{6x^2+5x \;/x^2}{-2x^2+5x-3 \;/x^2} = \lim_{x\to\infty} \frac{6+\cancel{5/x}^{\,0}}{-2+\cancel{\frac{5}{x}}^{\,0}-\cancel{\frac{3}{x^2}}^{\,0}} = \frac{6}{-2} = -3$ $\to$ $\boxed{y=-3}$ H. Asymp as $x\to\infty$; since $g(x)$ even, same limit as $x\to-\infty$.

④





⑤ $f(x) = \begin{cases} \sqrt{-x}, & x<0 \\ 3-x, & 0 \le x < 3 \\ (x-3)^2, & x \ge 3 \end{cases}$

must match at $x=0$: $f(0) = 3-0 = 3$, $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \sqrt{-x} = \sqrt{-0} = 0$; $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} 3-x = 3-0 = 3$ $\}$ $\lim_{x\to 0} f(x)$ does not exist $\boxed{\text{at } x=0 \text{ not continuous}}$

must match at $x=3$: $f(3) = (3-3)^2 = 0$ $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} 3-x = 3-3 = 0$; $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (x-3)^2 = (3-3)^2 = 0$ $\}$ $\lim_{x\to 3} f(x) = 0 = f(3)$ $\boxed{\text{at } x=3 \text{ continuous}}$

(continuous everywhere else)