



Show all work, including indications of mental steps, on the lined paper provided. Label and separate clearly (draw lines) each part of each problem and **BOX** each final short response requested (and nothing else). Use proper mathematical notation: "symbol" = "expression representing symbol" = Don't misuse equal signs (use " \rightarrow " or " $:$ " to indicate a relation which is not equality), but do connect equal expressions with equal signs. Give exact answers, not decimal approximations (unless requested). Nothing on this test may be justified or supported by technology output but you may use technology to check your work. This is a test about thinking, reasoning, and good communication of the process.

- ① A ball is thrown into the air so that its height h (in feet) after t seconds is given by $y = 40t - 16t^2 = f(t)$.
- What is the average velocity from $t=0$ to $t=1$? (i.e., average rate of change of y with respect to t over this interval).
 - Find the velocity $V(t) = f'(t)$ using the limit definition (state it and evaluate it).
 - Evaluate the velocity at $t=0$ and $t=2$, namely $V(0)$ and $V(2)$, using your previous result.
 - Find the time at which the velocity is zero and find the corresponding height (you may give the result as a decimal number).

- ② If $f(x) = x - \frac{2}{x}$, then $f'(x) = 1 + \frac{2}{x^2}$. Using this information, write the equation for the tangent line to the graph of f in the x - y plane at $x=-1$ and put into slope intercept form (solve it for y).

③ $g(x) = \frac{6x^2 + 5x}{(1-x)(2x-3)} = \frac{6x^2 + 5x}{-2x^2 + x - 3}$, $h(x) = \ln((1-x)(2x-3))$ (natural log)

- What is the domain of h ? Show some work to support your claim.
- Does g have vertical asymptotes? Write their equations and state and evaluate at least one limit to confirm that they represent vertical asymptotes, if present.
- Does g have horizontal asymptotes ($x \rightarrow \infty$ or $x \rightarrow -\infty$)? State and evaluate the appropriate limits and write the equations of any horizontal asymptotes you find for the graph $y = g(x)$.

- ④ Sketch the graph of an example of a function f that satisfies all of the following conditions:

$$\lim_{x \rightarrow 0^+} f(x) = -2, \quad \lim_{x \rightarrow 0^-} f(x) = 1, \quad f(0) = -1, \quad \lim_{x \rightarrow 2^-} f(x) = \infty, \quad \lim_{x \rightarrow 2^+} f(x) = -\infty,$$

$$\lim_{x \rightarrow \infty} f(x) = 3, \quad \lim_{x \rightarrow -\infty} f(x) = 4.$$

- ⑤ $f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ 3-x & \text{if } 0 \leq x < 3 \\ (x-3)^2 & \text{if } x \geq 3 \end{cases}$ Is f continuous at $x=0$ and $x=3$? Explain why or why not using limit notation.

After completing the exam, read and sign the following pledge, if it applies to you:

During this examination, all work has been my own and I have not opened any software other than MAPLE on my computer. I give my word as a decent human being that I have not resorted to any ethically questionable means of improving my performance or that of any one else on this examination, nor will I after I complete it.

Signature:

Date: October 11, 2006