higherorder linear homogeneous constant coefficient DEs

$y''' - y' = 0$ ——— 3 arb constants in soln → 3d solution space

$y = e^{rx}$ ↗ $(\underbrace{r^3 - r}) e^{rx} = 0$

$r(r^2-1) = 0$

$r = 0, 1, -1$

$e^{rx} = e^{0x}, e^{x}, e^{-x} \equiv y_1, y_2, y_3$ basis of soln space → so span 3-d solnspace

test for lin. ind.? $c_1 y_1 + c_2 y_2 + c_3 y_3 = 0$

↳ $c_1 y_1' + c_2 y_2' + c_3 y_3' = 0$

↳ $c_1 y_1'' + c_2 y_2'' + c_3 y_3'' = 0$

$$\underbrace{\begin{bmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{bmatrix}}_{W(y_1, y_2, y_3)} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

must be true for all x, any particular x — wronskian matrix

$\underbrace{\det W(y_1 y_2, y_3)|_{x=x_0}}_{\text{wronskian function}} \neq 0 \Leftrightarrow W(y_1, y_2, y_3)$ invertible for any x_0

only zero soln, [columns lin ind; $\{y_1, y_2, y_3\}$ lin ind.

IVP: $y = c_1 y_1 + c_2 y_2 + c_3 y_3 = c_1 + c_2 e^{x} + c_3 e^{-x}$

$y' = \ \dots \ = 0 + 1 c_2 e^{x} - 1 c_3 e^{-x}$

$y'' = \ \dots \ = 0 + 1 c_2 e^{x} + 1 c_3 e^{-x}$

$$\underbrace{\begin{bmatrix} y(0) \\ y'(0) \\ y''(0) \end{bmatrix}}_{\text{initial data vector}} = \underbrace{\begin{bmatrix} 1 & e^x & e^{-x} \\ 0 & e^x & -e^{-x} \\ 0 & e^x & e^{x} \end{bmatrix}}_{W(1, e^x, e^{-x})|_{x=0}} \Bigg|_{x=0} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} \rightarrow \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \left[W(1, e^x, e^{-x})|_{x=0} \right]^{-1} \begin{bmatrix} y(0) \\ y'(0) \\ y''(0) \end{bmatrix}$$

unique soln

← initial data vectors of lin. ind. solns are also linearly ind.

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nonhomogeneous case: $y''' - y' = 3\cos 2x$

↳ $y = y_h + y_p$, $y_h = c_1 + c_2 e^{x} + c_3 e^{-x}$

y_p = particular soln of DE.