# Word problem to math problem to solution

In a certain controlled environment the available food supply will support a maximum number $N$ of bacteria. Assume that the rate of growth of the number of bacteria is proportional to the difference between the maximum number and the number present.

dep variable "$n$" for number: $n(t)$

$\frac{dn}{dt} \propto (N-n)$

How long will it take for the population to reach 95% of the maximum number if initially the population was 25% of the maximum number $\to n(0) = .25N$ and doubles in the first week? $\to n(1) = 2n(0) = .5N$

independent variable: time $\to t$ = number of $\underbrace{\text{weeks}}_{\text{unit}}$

proportional — constant of proportionality

$A \propto B \to A = kB \longrightarrow$

$$\boxed{\frac{dn}{dt} = k(N-n)}$$ separable DEQ with unknown constant $k$.

$n(0) = .25N \leftrightarrow$ initial condition $\to$ determines constant

$n(1) = .5N \leftrightarrow$ extra condition $\to$ determines $k$

Goal: Find $n(t)$ then: $n(T) = .95N$, solve for $T$

solution

$$\int \frac{dn}{N-n} = \int k\,dt \to -\ln(N-n) = kt + C_1 \to \underbrace{e^{-\ln(N-n)}}_{\left[e^{\ln(N-n)}\right]^{-1} = \frac{1}{N-n}} = e^{kt+C_1} = e^{kt}e^{C_1}$$

$$\to N-n = \left(e^{C_1}e^{kt}\right)^{-1} = e^{-C_1}e^{-kt} = C_2 e^{-kt} \to \boxed{n = N - C_2 e^{-kt}} \quad \text{gen soln.}$$

$$.25N = n(0) = N - C_2e^0 = N - C_2 \to C_2 = N - .25N = .75N \to \boxed{n = N - .75Ne^{-kt} = N(1-.75e^{-kt})}$$

particular soln.:

$$\frac{.5\cancel{N}}{\cancel{N}} = \frac{n(1)}{N} = \frac{\cancel{N}(1-.75e^{-k})}{\cancel{N}} \to .5 = 1 - .75e^{-k} \to .75e^{-k} = 1 - .5 = .5$$

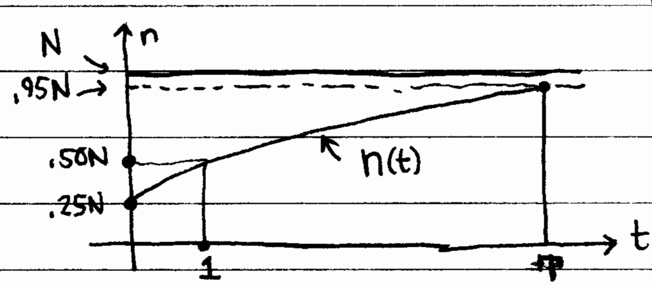
$$e^{-k} = \frac{.50}{.75} = \frac{2}{3}$$

$$\boxed{n = N(1-.75(e^{-k})^t) = N\left(1-.75\left(\tfrac{2}{3}\right)^t\right)} \quad \text{final solution}$$

$$\frac{.95\cancel{N}}{\cancel{N}} = \frac{n(T)}{N} = \frac{\cancel{N}\left(1-.75\left(\frac{2}{3}\right)^T\right)}{\cancel{N}} \to .95 = 1-.75\left(\tfrac{2}{3}\right)^T \to .75\left(\tfrac{2}{3}\right)^T = 1-.95 = .05$$

$$\to \left(\tfrac{2}{3}\right)^T = \frac{.05}{.75} = \frac{1}{15} \to \underbrace{\ln\left(\tfrac{2}{3}\right)^T}_{T\ln\left(\frac{2}{3}\right)} = \ln\frac{1}{15} = -\ln 15 \to \boxed{T = -\frac{\ln 15}{\ln(2/3)} = \frac{\ln 15}{\ln(3/2)}}$$

$\approx 6.67 \to 6.7$ (weeks)




"It will take approximately 6.7 weeks"

[word answer]