

driven constant coefficient linear DEs: problem completion

$$y'' + 4y' + 4y = e^{2x} + e^{-2x}$$

$$y_h = (c_1 + c_2 x) e^{-2x}$$

$$4 [y_p = c_3 x^2 e^{-2x} + c_4 e^{2x}]$$

backsubstitute preliminaries:

$$4 [y_p' = c_3 x^2 (-2e^{-2x}) + 2x c_3 e^{-2x} + 2c_4 e^{2x}]$$

$$4 [y_p'' = c_3 x^2 (-2)^2 e^{-2x} + 2x c_3 (-2e^{-2x}) + 2x c_3 (-2e^{-2x}) + 2c_3 e^{-2x} + 4c_4 e^{2x}]$$

$$y_p'' + 4y_p' + 4y_p = c_3 x^2 e^{-2x} (4 - 8 + 4) + 2c_3 x e^{-2x} (4 - 2 - 2) + (4 + 8 + 4)c_4 e^{2x}$$

$$= \underbrace{2c_3}_{=1} e^{-2x} + \underbrace{16c_4}_{\text{LHS}} e^{2x} \stackrel{\text{set equal}}{=} e^{2x} + e^{-2x} \quad \text{RHS}$$

$$c_3 = \frac{1}{2}, c_4 = \frac{1}{16}$$

$$y_p = \frac{1}{2} x^2 e^{-2x} + \frac{1}{16} e^{2x}$$

all the terms with x or x^2 times an exponential must drop out & they do.

[this is easy to show in general]

$$y = y_h + y_p = (c_1 + c_2 x + \frac{1}{2} x^2) e^{-2x} + \frac{1}{16} e^{2x}$$

$$y' = (-2)(c_1 + c_2 x + \frac{1}{2} x^2) e^{-2x} + (c_2 + x) e^{-2x} + \frac{1}{8} e^{2x}$$

$$y(0) = c_1 + \frac{1}{16} = 2$$

$$c_1 = 2 - \frac{1}{16} = \frac{31}{16}$$

$$y'(0) = -2c_1 + c_2 + \frac{1}{8} = 0$$

$$c_2 = 2c_1 - \frac{1}{8} = \frac{31}{8} - \frac{1}{8} = \frac{30}{8} = \frac{15}{4}$$

extra constant terms, but otherwise same as before

$$y = \left(\frac{31}{16} + \frac{15}{4} x + \frac{1}{2} x^2 \right) e^{-2x} + \frac{1}{16} e^{2x}$$

done.