

## driven constant coefficient linear DEs (method of undetermined coefficients)

$$\underbrace{y'' + 4y' + 4y = e^x + e^{-x}}_{(D^2 + 4D + 4)y}, \quad y(0) = 2, \quad y'(0) = 0$$

$$\underbrace{(D^2 + 4D + 4)y}_{r^2 + 4r + 4 = 0}$$

$$(r+2)^2 = 0$$

$$r = -2 \quad \text{mult: 2}$$

$$y_h = (C_1 + C_2x)e^{-2x}$$

homogeneous soln.

$y = y_h + y_p$  need to find  
particular soln  $y_p$

$f(x)$  is itself a soln of a const coeff linear hom DE:

$$\underbrace{e^x + e^{-x}}_{r=1 \quad r=-1} \rightarrow (r-1)(r+1) = 0$$

$$(D-1)(D+1)f(x) = 0$$

$$(D-1)(D+1)[(D+2)^2 y = e^x + e^{-x}]$$

$$(D-1)(D+1)(D+2)^2 y = (D-1)(D+1)(e^x + e^{-x}) = 0$$

4th order hom DE.

$$r = -2, 1, -1$$

$$\text{mult: } 2, 1, 1$$

$$4\text{th order DE gensln: } y = \underbrace{(C_1 + C_2x)e^{-2x}}_{y_h} + \underbrace{C_3 e^x + C_4 e^{-x}}_{y_p}$$

not a soln of 2nd order D.E., backsub  $y_p$  to fix constants  
to make it a soln.

$$1 (y_p = C_3 e^x + C_4 e^{-x})$$

$$4 (y_p' = C_3 e^x - C_4 e^{-x})$$

$$1 (y_p'' = C_3 e^x + C_4 e^{-x})$$

$$y_p'' + 4y_p' + 4y_p = (4+4+1)C_3 e^x + (4-4+1)C_4 e^{-x} \\ = \underbrace{9C_3 e^x}_{1} + \underbrace{C_4 e^{-x}}_{1} = e^x + e^{-x} \rightarrow C_3 = \frac{1}{9}, C_4 = 1 \rightarrow y_p = \frac{1}{9}e^x + e^{-x}$$

$$y = (C_1 + C_2x)e^{-2x} + \frac{1}{9}e^x + e^{-x}$$

gensln of 2nd order DE.

solve ICS

$$y = (C_1 + C_2x)e^{-2x} + \frac{1}{9}e^x + e^{-x}$$

$$y' = [C_2 + (-2)(C_1 + C_2x)]e^{-2x} + \frac{1}{9}e^x - e^{-x}$$

$$y(0) = C_1 + \frac{1}{9} + 1 = 2 \rightarrow C_1 = \frac{8}{9}$$

$$y'(0) = -2C_1 + C_2 + \frac{1}{9} - 1 = 0 \rightarrow C_2 = \frac{8}{9} + 2(\frac{8}{9}) \\ = 3(\frac{8}{9})$$

$$y = \frac{8}{9}(1+3x)e^{-2x} + \frac{1}{9}e^x + e^{-x} \quad \text{IVP soln}$$

complication: some new roots coincide with some old roots

$$\underbrace{y'' + 4y' + 4y = e^{2x} + e^{-2x}}_{(D+2)^2 y \text{ same as before}} \quad y(0) = 2, \quad y'(0) = 0$$

$$(D-2)(D+2)(D+2)^2 y = 0 \quad (D-2)(D+2)(e^{2x} + e^{-2x}) = 0$$

$$(r-2)(r+2)^3 = 0$$

$$r=2 \quad r=-2 \quad \text{mult: 3}$$

$$y = (C_1 + C_2x + C_3x^2)e^{-2x} + C_4 e^{2x}$$

$$y_p = C_3 x^2 e^{-2x} + C_4 e^{2x}$$

HW: now backsub  $y_p$  into original DE to determine  $C_3, C_4$ ; then solve ICS.

\* CHECK WITH dsolve

Note:  $e^{-2x}$  satisfies hom. DE

\*  $x \cdot x e^{-2x}$  satisfies hom. DE

\*  $x^2 \cdot x^2 e^{-2x}$  does not, ok