

3/2 power deceleration: what can we learn?

$$\frac{dV}{dt} = -kV^{3/2}, \quad V(0) = V_0 \quad \longrightarrow \quad \lim_{t \rightarrow \infty} \Delta x \text{ is finite stopping distance.}$$

The whole point of this problem is that although the velocity takes an infinite time to go to zero, the total distance traveled is finite. Calculating this distance and getting an idea of the time scale over which it gets "pretty close" to stopping should be the goal.

separating and integrating one finds:

$$V = \frac{V_0}{\left(1 + \frac{1}{2} k V_0^{1/2} t\right)^2} = \frac{V_0}{(1 + t/\tau)^2}$$

must be dimensionless so define

$$\frac{1}{2} k V_0^{1/2} t \equiv \frac{t}{\tau}$$

$\tau = \frac{2}{k V_0^{1/2}}$

time scale for this process

notice $V_0 \tau = \frac{2}{k} V_0^{1/2} = D$

Integrating one finds the displacement

$$\begin{aligned}\Delta x &= x - x_0 = \int_0^t v(t) dt = \underbrace{\frac{2v_0^{1/2}}{k} \left(1 - \frac{1}{1 + \frac{k}{2} v_0^{1/2} t} \right)}_{\equiv D = \frac{2v_0^{1/2}}{k} \text{ stopping distance since}} \\ &= D \left(1 - \frac{1}{1 + t/\tau} \right) = D \left(\frac{1 + t/\tau - 1}{1 + t/\tau} \right) \quad \lim_{t \rightarrow \infty} \Delta x = D \\ &= \frac{D(t/\tau)}{(1 + t/\tau)} = D \frac{1}{(1 + \frac{v}{\tau})} \approx D \left(1 - \frac{v}{\tau} \right) \quad (\text{Taylor series approx} \\ &\qquad\qquad\qquad \text{= linear approx})\end{aligned}$$

The time scale is the time it would take to reach the stopping distance at the constant initial velocity. The ratio $\frac{t}{\tau}$ is then exactly the fraction of the stopping distance still to go at time t (when $t \gg \tau$).
 The DE can be written in dimensionless variables (exercise) as:

$$\frac{du}{dT} = -u^{3/2}, \quad u \equiv \frac{v}{v_0}, \quad T \equiv t/\tau$$