$\boxed{5.3}$ $\quad y''+6y'+13y=0,\quad y(0)=2,\ y'(0)=-4$

$y=e^{rx} \to (r^2+6r+13)\,e^{rx}=0$

$\neq 0 \longrightarrow r=\dfrac{-6\pm\sqrt{36-4(13)}}{2}=\dfrac{-6\pm\sqrt{-16}}{2}=-3\pm 2i$

$e^{rx}=e^{(-3\pm 2i)x}=e^{-3x}e^{\pm 2ix}=e^{-3x}(\cos 2x\pm i\sin 2x)\to$ complex conj basis

$\hookrightarrow e^{-3x}\cos 2x,\ e^{-3x}\sin 2x \leftarrow$ real basis

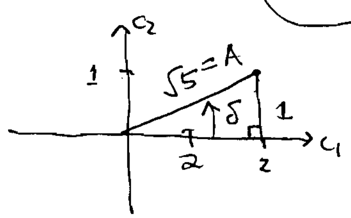
$y=c_1e^{-3x}\cos 2x+c_2e^{-3x}\sin 2x = e^{-3x}(c_1\cos 2x+c_2\sin 2x)$ general soln

$y'=-3e^{-3x}(c_1\cos 2x+c_2\sin 2x)+e^{-3x}(-2c_1\sin 2x+2c_2\cos 2x)$

$y(0)=c_1 \qquad =2$

$y'(0)=-3c_1+2c_2=-4 \to c_2=\dfrac{-4+3(2)}{2}=1$

$\boxed{y= e^{-3x}(2\cos 2x+\sin 2x) = \sqrt{5}\,e^{-3x}\cos\left(2x-\arctan\tfrac12\right)}$ IVP solution



$\delta=\arctan\frac12$ (exact)

≈ 0.4636 rad (approximate)

$\approx 26.6°$

≈ 0.074 cycles (fraction of circle) $\}$ interpretation

The peaks lag about 27° behind (later in x) than the unshifted cosine. Initial amplitude is $\sqrt5\approx 2.27$.

$2\Delta x=\delta$

$\Delta x=\dfrac{\delta}{2}=\dfrac{0.4636}{2}=0.2318$

so soln curve touches envelope (top curve) first at $x\approx 0.2318$.

(See Maple plot worksheet.)