2nd Order linear homogeneous DEs with complex exponential solns

$y'' + 6y' + 13y = 0$

$y = e^{rx} \rightarrow \underbrace{(r^2+6r+13)}_{=0} e^{rx} = 0$

$r = \cdots = -3 \pm 2i$

$e^{rx} = e^{(-3\pm 2i)x} = e^{-3x} e^{\pm 2ix}$

$= e^{-3x}(\cos 2x \pm i \sin 2x)$

complex conjugate pair

linear change of basis of soln space

old soln basis $\qquad$ new soln basis

$$\begin{bmatrix} e^{(-3+2i)x} \\ e^{(-3-2i)x} \end{bmatrix} = \begin{bmatrix} 1 & i \\ 1 & -i \end{bmatrix} \begin{bmatrix} e^{-3x}\cos 2x \\ e^{-3x}\sin 2x \end{bmatrix}$$

$$\begin{bmatrix} z \\ \bar z \end{bmatrix} = \begin{bmatrix} 1 & i \\ 1 & -i \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$ relation between z and its Re and Im parts!

$y = c_1 e^{(-3+2i)x} + c_2 e^{(-3-2i)x}$ $\rightarrow$ $c_1 = \bar c_2$, $c_2 = \bar c_1 \equiv \dfrac{C_1 + iC_2}{2}$

$= \bar y = \bar c_1 e^{(-3-2i)x} + \bar c_2 e^{(-3+2i)x}$

$= \dfrac{C_1 - iC_2}{2} e^{-3x}(\cos 2x + i\sin 2x) + \dfrac{C_1+iC_2}{2} e^{-3x}(\cos 2x - i \sin 2x)$

$= \frac{1}{2} e^{-3x}(C_1\cos 2x + C_2 \sin 2x + \cancel{i(C_1\sin 2x - C_2\cos 2x)})$ $\quad = \boxed{e^{-3x}(C_1\cos 2x + C_2\sin 2x)}$

$+ \frac{1}{2} e^{-3x}(C_1\cos 2x + C_2\sin 2x - i(C_1 \sin 2x - C_2\cos 2x))$

0

$$\text{span}\{e^{(-3+2i)x}, e^{(-3-2i)x}\} = \text{span}\{e^{-3x}\cos 2x, e^{-3x}\sin 2x\}$$

$\boxed{\text{SKIP complex arithmetic}}$, go directly to using new explicitly real basis expression for general soln

in general:

complex conjugate basis $\{e^{(a+ib)x}, e^{(a-ib)x}\}$	$\longrightarrow$	explicitly real basis $\{e^{ax}\cos bx, e^{ax}\sin bx\}$

initial conditions: $y(0) = 2$, $y'(0) = -4$

$y = e^{-3x}(C_1\cos 2x + C_2 \sin 2x)$

$y' = -3e^{-3x}(C_1\cos 2x + C_2\sin 2x)$

$\quad + e^{-3x}(-2C_1\sin 2x + 2C_2\cos 2x)$

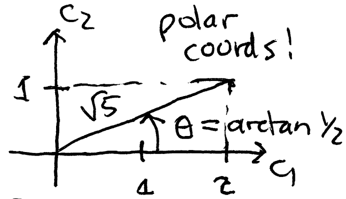
$y(0) = C_1 = 2$

$y'(0) = -3C_1 + 2C_2 = -4 \rightarrow C_2 = \frac{1}{2}(3(2)-4) = 1$

$\boxed{y = e^{-3x}(2\cos 2x + \sin 2x)}$

$(C_1, C_2) = (2, 1)$

$(A, \delta) = (\sqrt{5}, \arctan \frac{1}{2})$

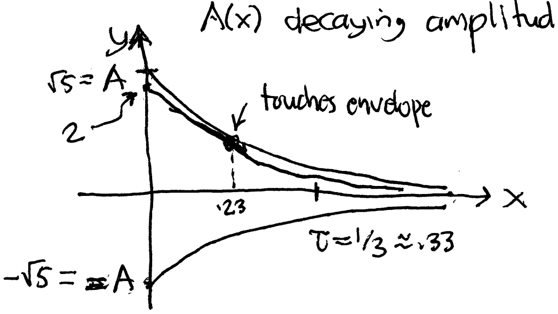


$= \underbrace{\sqrt{5}e^{-3x}}_{A(x)\text{ decaying amplitude}} \cos(2x - \arctan \frac{1}{2})$

$C_1 \cos\omega x + C_2 \sin \omega x = A\cos(\omega x - \delta)$

phaseshifted cosine form of sinusoidal factor

period $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$



don't see much of oscillation

decays too quickly