

RREF = Row Reduced Echelon Form : Gauss-Jordan row reduction algorithm

row operations: SwapRow MultiplyRow AddRow
 $R_1 \leftrightarrow R_3$ $R_2 \rightarrow 5R_2$ $R_2 \rightarrow R_2 + 3R_1$
 (swap rows (multiply row 2 (add to row 2 : 3 times row 1)
 1 and 3) by 5)

1. Start at the (1,1) entry.

- restart: 2. If entry zero, swap up a row from below with a nonzero entry in this column and proceed to the next step (unless there are no nonzero entries below it, then move over right to next entry and restart here). [SWAPROW]
3. If entry nonzero, divide its row by this entry to make it 1. [MULTIPLY ROW]
4. Pivot on this entry ("the pivot entry") to make the remaining column entries zero (by adding multiples of this "pivot row" to the other rows with coefficients equal to minus the column entries to be made zero). [ADDROW]
5. Move diagonally down to the next entry (over one, down one) and restart.
6. Continue until there is no next entry to move to according to the procedure.

Terminology: The leading entry of a row is the first nonzero entry from left to right.

The resulting unique reduced matrix is characterized by:

1. The leading entries are 1's which move down and to the right.
[Equivalently: the leading entry in each row is to the right of the previous one above.]
2. There are 0's above and below each leading entry.
3. Any zero rows are at the bottom.

If no zero entries are encountered in the pivot positions which cannot be "swapped up" to a nonzero value, then one just pivots down the main diagonal until hitting the bottom or right edge of the matrix and stopping.

Row echelon form differs from reduced row echelon form in that one does partial pivoting, eliminating entries below but not above leading ones, and one skips the MultiplyRow operation to make leading entries 1. This is called Gaussian elimination instead of Gauss-Jordan elimination. We always want to do a complete reduction to RREF form.

EXAMPLE 1: 2x2

$$\frac{1}{2} (10x + 6y = 2)$$

$$x - 2y = 8$$

$$5x + 3y = 1$$

$$x - 2y = 8$$

$$* (5) \begin{cases} x - 2y = 8 \\ 5x + 3y = 1 \\ \hline - (5x - 10y = 40) \end{cases}$$

$$x - 2y = 8$$

$$\frac{1}{13} (0x + 13y = -39)$$

$$*2 \begin{cases} 0x + 2y = -6 \\ x - 2y = 8 \\ \hline (0x + 1y = -3) \end{cases}$$

$$1x + 0y = 2$$

$$0x + 1y = -3$$

soln: $x = 2$
 $y = -3$

$$\begin{bmatrix} 10 & 6 & 2 \\ 1 & -2 & 8 \end{bmatrix} \leftarrow R_1 \rightarrow \frac{1}{2} R_1$$

"Multiply Row"

$$\begin{bmatrix} 5 & 3 & 1 \\ 1 & -2 & 8 \end{bmatrix} \swarrow R_1 \leftrightarrow R_2$$

"Swap Row"

$$\begin{bmatrix} 1 & -2 & 8 \\ 5 & 3 & 1 \\ -5 & +10 & -40 \end{bmatrix} \leftarrow R_2 \rightarrow R_2 - 5R_1$$

"Add Row"

eliminate x from 2nd eqn

$$\begin{bmatrix} 1 & -2 & 8 \\ 0 & 13 & -39 \end{bmatrix} \leftarrow R_2 \rightarrow \frac{1}{13} R_2$$

$$\begin{matrix} +0 & +2 & -6 \\ \begin{bmatrix} 1 & -2 & 8 \\ 0 & 1 & -3 \end{bmatrix} \end{matrix} \uparrow R_1 \rightarrow R_1 + 2R_2$$

eliminate y from 1st eqn

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \end{bmatrix}$$

EXAMPLE 2: 3x3

$$3x + 8y + 7z = 20$$

$$x + 2y + z = 4$$

$$2x + 7y + 9z = 23$$

EXAMPLE 3: 3x5

$$-2x_1 + 4x_2 - 6x_3 - 3x_4 - 6x_5 = -13$$

$$x_1 - 2x_2 + 3x_3 + 2x_4 + x_5 = 10$$

$$3x_1 - 6x_2 + 6x_3 + 2x_5 = -3$$