

driven RLC circuit

$R = 16 \text{ ohms}$, $C = \frac{1}{40} \text{ farads} = 0.025 \text{ farads} = 25 \text{ millifarads}$, $L = 8 \text{ henries}$

$E(t) = 4 \sin 2t \text{ volts}$, $\omega = 2 \text{ rad/sec} = \frac{2}{2\pi} \approx 0.318 \text{ Hz}$ (cycles/sec)

$$LI'' + RI' + \frac{1}{C}I = E'(t), \quad I(0) = I'(0) = 0 \quad \text{starting with no current flow}$$

$$8I'' + 16I' + 40I = 8 \cos 2t$$

$$[I'' + 2I' + 5I = \cos 2t] \rightarrow R_0 = 2, T_0 = \frac{1}{2}, \omega_0 = \sqrt{5} \approx 2.24 \text{ rad/sec} \approx 0.356 \text{ Hz}$$

$$(D^2 + 2D + 5)I = \cos 2t \longrightarrow \omega = 2 \rightarrow r = \pm 2i \quad (D^2 + 4) \cos 2t = 0$$

$$I = e^{rt} \quad r^2 + 2r + 5 = 0 \\ r = -2 \pm \frac{\sqrt{4-4 \cdot 5}}{2} = -1 \pm 2i$$

$$I_h = e^{-t}(c_1 \cos 2t + c_2 \sin 2t)$$

$$(D^2 + 4)(D^2 + 2D + 5)I = (D^2 + 4)\cos 2t = 0$$

$$r = \pm 2i \quad r = -1 \pm 2i$$

$$I = \underbrace{e^{-t}(c_1 \cos 2t + c_2 \sin 2t)}_{I_h} + \underbrace{c_3 \cos 2t + c_4 \sin 2t}_{I_p}$$

$$5 [I_p = c_3 \cos 2t + c_4 \sin 2t]$$

$$2 [I_p' = -2c_3 \sin 2t + 2c_4 \cos 2t]$$

$$1 [I_p'' = -4c_3 \cos 2t - 4c_4 \sin 2t]$$

$$I_p'' + 2I_p' + 5I_p = [(5-4)c_3 + 4c_4] \cos 2t + [-4c_3 + (5-4)c_4] \sin 2t$$

$$= \underbrace{(c_3 + 4c_4) \cos 2t}_{=1} + \underbrace{(-4c_3 + c_4) \sin 2t}_{=0} = \cos 2t$$

$$\begin{bmatrix} 1 & 4 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} c_3 \\ c_4 \end{bmatrix} = \frac{1}{17} \begin{bmatrix} 1 & -4 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/17 \\ 4/17 \end{bmatrix}$$

$$I = e^{-t}(c_1 \cos 2t + c_2 \sin 2t) + \frac{1}{17}(\cos 2t + 4 \sin 2t)$$

$$I' = -e^{-t}(c_1 \cos 2t + c_2 \sin 2t) + \frac{1}{17}(-2 \sin 2t + 8 \cos 2t) \\ + e^{-t}(-2c_1 \sin 2t + 2c_2 \cos 2t)$$

$$I(0) = c_1 + \frac{1}{17} = 0 \rightarrow c_1 = -1/17$$

$$I'(0) = -c_1 + 2c_2 + \frac{8}{17} = 0 \rightarrow c_2 = \frac{1}{2} \left(-\frac{8}{17} + \left(-\frac{1}{17} \right) \right) = -9/34$$

$$I = \underbrace{-\frac{1}{34}e^{-t}(2 \cos 2t + 9 \sin 2t)}_{\text{transient } T=1, 5T=5 \text{ sec}} + \underbrace{\frac{1}{17}(\cos 2t + 4 \sin 2t)}_{\frac{1}{17} \cos(2t - \delta)}$$

"steady state"

see Maple plots online

