

resonance calculation (2) (back to example)

$$y'' + 4y' + 5y = B_0 \cos \omega t$$

$$\S [y_p = c_3 \cos \omega t + c_4 \sin \omega t]$$

$$\uparrow [y_p' = -\omega c_3 \sin \omega t + \omega c_4 \cos \omega t]$$

$$\downarrow [y_p'' = -\omega^2 c_3 \cos \omega t - \omega^2 c_4 \sin \omega t]$$

$$y_p'' + 4y_p' + 5y_p = [(5-\omega^2)c_3 + 4\omega c_4] \cos \omega t \\ + [-4\omega c_3 + (5-\omega^2)c_4] \sin \omega t \\ = B_0 \cos \omega t$$

$$(5-\omega^2)c_3 + 4\omega c_4 = B_0$$

$$-4\omega c_3 + (5-\omega^2)c_4 = 0$$

$$\begin{bmatrix} 5-\omega^2 & 4\omega \\ -4\omega & 5-\omega^2 \end{bmatrix} \begin{bmatrix} c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} B_0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} c_3 \\ c_4 \end{bmatrix} = \frac{1}{(5-\omega^2)^2 + 16\omega^2} \begin{bmatrix} 5-\omega^2 & -4\omega \\ 4\omega & 5-\omega^2 \end{bmatrix} \begin{bmatrix} B_0 \\ 0 \end{bmatrix} \\ = \frac{B_0}{(5-\omega^2)^2 + 16\omega^2} \begin{bmatrix} 5-\omega^2 \\ 4\omega \end{bmatrix}$$

steady state soln:

$$y_p = \frac{B_0}{(5-\omega^2)^2 + 16\omega^2} [(5-\omega^2) \cos \omega t + 4\omega \sin \omega t]$$

amplitude response function:

$$A(\omega) = \frac{B_0 \sqrt{(5-\omega^2)^2 + 16\omega^2}}{(5-\omega^2)^2 + 16\omega^2} = \frac{B_0}{[(5-\omega^2)^2 + 16\omega^2]^{1/2}} \\ = B_0 [\dots]^{-1/2}$$

peak:

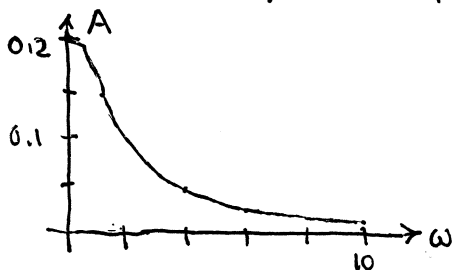
$$0 = A'(\omega) = -\frac{1}{2} [(5-\omega^2)^2 + 16\omega^2]^{-3/2} [2(5-\omega^2)(-2\omega) + 32\omega]$$

$$\omega^2 = -4 \quad \text{no real solution,} \\ \text{no peak}$$

$$\text{only slightly underdamped: } Q = \frac{\sqrt{5}}{4} \approx 0.56 > 0.5,$$

$$\text{need } Q > \frac{1}{\sqrt{2}} \approx 0.707 \text{ for peak to appear,}$$

no resonance, biggest response at zero frequency



you don't need to remember any formulas apart from the amplitude relation:

$$c_1 \cos \omega t + c_2 \sin \omega t$$

$$\hookrightarrow (c_1, c_2) \rightarrow A = \sqrt{c_1^2 + c_2^2}$$