

CHECKING SOLUTIONS OF EQUATIONS

solve an equation for an "unknown": $x^2 - 3x + 2 = 0$ $\xrightarrow[\text{technique}]{\text{soln}}$ $\begin{cases} \text{general soln} \\ \text{---} \\ x=1, x=2 \end{cases}$

a particular soln

check the solution by substituting it back into the equation & simplifying LHS, RHS independently:

$$\begin{array}{ll} (1)^2 - 3(1) + 2 = 0 & (2)^2 - 3(2) + 2 = 0 \\ 1 - 3 + 2 = 0 & 4 - 6 + 2 = 0 \\ 0 = 0 \checkmark & 0 = 0 \checkmark \end{array}$$

solve a differential equation for an "unknown": the variable function

the solution is an equation expressing the unknown (LHS) as a function of the independent variable (RHS)

$$1) \frac{dy}{dx} = ky \quad \begin{array}{l} \text{unknown} \\ \text{---} \\ \text{parameter in DE} \end{array} \quad y = Ce^{kx} \quad \begin{array}{l} \text{---} \\ \text{"arbitrary constant"} \\ \text{(any real number)} \end{array}$$

$y = 2e^{kx}$ is a particular soln
 $y = Ce^{kx}$ is the general soln

→ check: $\frac{d}{dx}(Ce^{kx}) = k(Ce^{kx})$ "backsubstitution" everywhere in eqn for unknown

$$\begin{aligned} C \frac{d}{dx} e^{kx} &= kC e^{kx} \\ C k e^{kx} &= C k e^{kx} \checkmark \end{aligned}$$

$$2) \frac{d^2y}{dx^2} + \omega^2 y = 0 \quad \begin{array}{l} \text{note 2} \\ \text{---} \\ \text{frequency } \omega > 0 \end{array} \quad y_1 = C_1 \cos \omega x + C_2 \sin \omega x \quad \begin{array}{l} \text{---} \\ \text{2 arbitrary constants} \end{array} \quad \text{"general soln"}$$

$y = 2 \cos \omega x + 4 \sin \omega x$ is a particular soln

→ check:

$$\begin{aligned} \frac{d^2}{dx^2}(C_1 \cos \omega x + C_2 \sin \omega x) + \omega^2(C_1 \cos \omega x + C_2 \sin \omega x) &= 0 \\ \frac{d}{dx}(C_1 \frac{d}{dx}(\cos \omega x) + C_2 \frac{d}{dx}(\sin \omega x)) + \omega^2 C_1 \cos \omega x + \omega^2 C_2 \sin \omega x &= 0 \\ \frac{d}{dx}(-C_1 \omega \sin \omega x + C_2 \omega \cos \omega x) + C_1 \omega^2 \cos \omega x + C_2 \omega^2 \sin \omega x &= 0 \\ -C_1 \frac{d}{dx} \sin \omega x + C_2 \frac{d}{dx} \cos \omega x + C_1 \omega^2 \cos \omega x + C_2 \omega^2 \sin \omega x &= 0 \\ -C_1 \omega^2 \cos \omega x - C_2 \omega^2 \sin \omega x + C_1 \omega^2 \cos \omega x + C_2 \omega^2 \sin \omega x &= 0 \\ 0 = 0 \checkmark \end{aligned}$$

memorize these 2 DE solutions. we will derive their solutions later but will use them throughout the semester: exponentials describe growth ($k > 0$) and decay ($k < 0$) while cosines & sines describe oscillations. (of frequency $\omega > 0$).