mixing tank DE (solved as a function of the model parameters)

$$\frac{dx}{dt} + \frac{r_o}{\underbrace{V_0+(r_i-r_o)t}_{V(t)\,\ge\, 0}}\,x = r_i c_i \qquad x(0) = x_0$$

$$e^{\int \frac{r_o}{V_0+(r_i-r_o)t}\,dt} = e^{\frac{r_o}{r_i-r_o}\ln|V_0+(r_i-r_o)t|} \quad \text{if } r_i-r_o \neq 0 \quad \xrightarrow{r_i=r_o=r} \quad e^{\int \frac{r_o}{V_0}dt} = e^{\frac{r_o}{V_0}t}$$

$$= (V_0+(r_i-r_o)t)^{\frac{r_o}{r_i-r_o}} = V^{\frac{r_o}{r_i-r_o}}$$

$$\frac{d}{dt}\left(V^{\frac{r_o}{r_i-r_o}}x\right) = r_i c_i (V_0+(r_i-r_o)t)^{\frac{r_o}{r_i-r_o}}$$

$$x V^{\frac{r_o}{r_i-r_o}} = \frac{r_i c_i}{r_i-r_o}\,\frac{V^{\frac{r_o}{r_i-r_o}+1}}{\frac{r_o}{r_i-r_o}+1} + C_1$$

$$\frac{\cancel{r_o}+r_i-\cancel{r_o}}{r_i-r_o} = \frac{r_i}{r_i-r_o}$$

$$= c_i V^{\frac{r_i}{r_i-r_o}} + C_1$$

$$x = c_i V^{\frac{r_i}{r_i-r_o}-\frac{r_o}{r_i-r_o}} + C_1 V^{-\frac{r_o}{r_i-r_o}}$$

$$= c_i V + C_1 V^{-\frac{r_o}{r_i-r_o}}$$

$$x_0 = c_i V_0 + C_1 V_0^{-\frac{r_o}{r_i-r_o}}$$

$$C_1 = (x_0 - c_i V_0)\,V_0^{\frac{r_o}{r_i-r_o}}$$

$$x = c_i V + (x_0 - c_i V_0)\,V^{-r_o/(r_i-r_o)}\,V_0^{\frac{r_o}{r_i-r_o}}$$

$$\boxed{x = c_i V + (x_0 - c_i V_0)\left(\frac{V}{V_0}\right)^{-\frac{r_o}{r_i-r_o}}}$$

$$c_o = \frac{x}{V} = c_i + \frac{x_0 - c_i V_0}{V}\left(\frac{V}{V_0}\right)^{-\frac{r_o}{r_i-r_o}}$$

$$= c_i + \frac{x_0 - c_i V_0}{V_0\left(\frac{V}{V_0}\right)}\left(\frac{V}{V_0}\right)^{-\frac{r_o}{r_i-r_o}}$$

$$= c_i + \underbrace{(c_o(0) - c_i)}_{\text{if } \neq 0}\underbrace{\left(\frac{V}{V_0}\right)^{-\left(\frac{r_o}{r_i-r_o}+1\right)}}_{} = c_i + (c_o(0)-c_i)\left(\frac{V}{V_0}\right)^{-\frac{r_i}{r_i-r_o}}$$

this forces it to zero if $r_i > r_o$ (increasing V, negative power) but also if $r_i < r_o$ (V decreasing, but power > 0)

outgoing concentration asymptotes to incoming concentration

$$\frac{d}{dt}\left(e^{\frac{r_o}{V_0}t}x\right) = r_i c_i e^{\frac{r_o}{V_0}t}$$

$$x e^{\frac{r_o}{V_0}t} = r_i c_i \frac{e^{\frac{r_o}{V_0}t}}{r_o/V_0} + C_1$$

$$= c_i V_0 e^{\frac{r_o}{V_0}t} + C_1$$

$$x = c_i V_0 + C_1 e^{-\frac{r_o}{V_0}t}$$

$$x_0 = c_i V_0 + C_1$$

$$C_1 = x_0 - c_i V_0$$

$$\boxed{x = c_i V_0 + (x_0 - c_i V_0)\,e^{-\frac{r_o}{V_0}t}}$$

$$c_o = \frac{x}{V_0} = c_i + \frac{x_0 - c_i V_0}{V_0}e^{-\frac{r_o}{V_0}t}$$

$$= c_i + \underbrace{(c_o(0) - c_i)}_{\text{if } \neq 0}\,\underbrace{e^{-\frac{r_o}{V_0}t}}_{\text{this forces it to zero}}$$