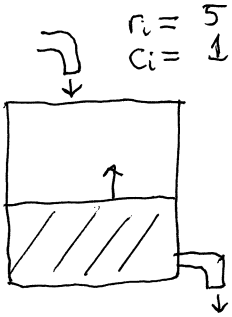
E&P3 1.5.37 mixing tank problem– increasing volume



$r_i = 5$ gal brine/s
$c_i = 1$ lb salt/(gal brine)

$x(t) =$ lbs salt

$r_o = 3$ gal brine/s

$V_{max} = 400$ gal

$c_o(t) = \frac{x(t)}{V(t)}$

$V_0 = 100$ gal brine, $x_0 = 50$ lb salt $\Rightarrow c_0(0) = \frac{50 \text{ lb salt}}{100 \text{ gal brine}} = 0.5$ lb salt/(gal brine)

volume: $V = V_0 + (r_i - r_o)t = 400$ filled tank (stop!)

$= 100 + 2t \rightarrow 2t = 300$

$t = 150$ s (2.5 min)

$V(150) = 400$!

$$\frac{dx}{dt} + \frac{r_o}{V_0 + (r_i - r_o)t} x = r_i c_i \xrightarrow{\text{plugin numbers}} \frac{dx}{dt} + \frac{3}{100+2t} x = 5$$

$$e^{\int \frac{3}{100+2t}\,dt} = e^{\frac{3}{2}\int \frac{2dt}{2t+100}} = e^{\frac{3}{2}\ln(100+2t)} = (100+2t)^{3/2}$$

$$(100+2t)^{3/2}\left(\frac{dx}{dt} + \frac{3}{100+2t}x\right) = 5(100+2t)^{3/2}$$

$$\frac{d}{dt}\left(x(100+2t)^{3/2}\right)$$

$$x(100+2t)^{3/2} = 5\int (100+2t)^{3/2}\,dt = \frac{5}{2}\int (100+2t)^{3/2}\,2dt$$

$$\int u^{3/2}\,du = \frac{u^{5/2}}{5/2} + \cdots$$

$$= \frac{5}{2}\frac{(100+2t)^{5/2}}{5/2} + C$$

$$= (100+2t)^{5/2} + C$$

$$x = (100+2t)^{-3/2}\left((100+2t)^{5/2} + C\right) = (100+2t) + C(100+2t)^{-3/2}$$

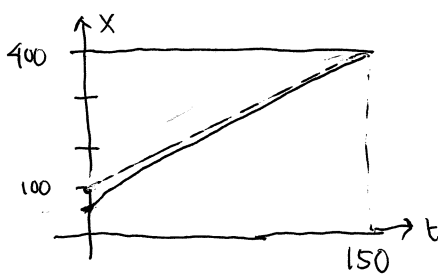
$$50 = x(0) = 100 + \frac{C}{100^{3/2}} = 100 + \frac{C}{1000} \rightarrow C = -50 \cdot 1000 = -5\times 10^4.$$

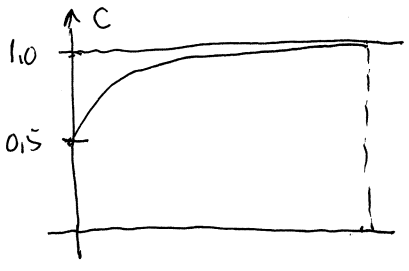
$$x = (100+2t) - \frac{5\times 10^4}{(100+2t)^{3/2}}$$

$$x(150) = 400 - \frac{5\times 10^4}{400^{3/2}} = 400 - \frac{5\times 10^4}{8000} = 400 - \frac{5}{800} \approx \boxed{393.75 \text{ lb salt}}$$

$400^{3/2} = 20^3 = 8\times 10^6$

$$c(150) \approx \frac{393.75}{400} \approx \boxed{0.984 \text{ lb/gal}}$$ nearly doubled from initial 0.5 lb/gal





← 1 lb/gal is asymptote (incoming concentration)

left graph dashed line: if had begun with $x(0) = 100$ concentration would not change, but x would increase linearly