

MATRIX MULTIPLICATION

is just an organized way of displaying all possible dot products between one set of vectors (rows on left!) and another set of vectors (columns on right!). Rows are ordered top to bottom, columns left to right

$$[A B]_{ij} = \underbrace{A}_{m \times n} \underbrace{B}_{n \times p} = \begin{bmatrix} R_1 \\ \vdots \\ R_m \end{bmatrix} \begin{bmatrix} C_1 \cdots C_p \end{bmatrix} = [R_i \cdot C_j]$$

rows on left columns on right

\vec{R}_i, \vec{C}_j both n -component vectors so can take their dot product to get a number

each column (fix j) of the right factor multiplies each row of the left factor by dot producting to produce the entries in the product matrix in that same column label j

example $\{ \langle 1, 2, 3 \rangle, \langle 3, 2, -1 \rangle, \langle -1, 4, 2 \rangle \}$ and $\{ \langle 2, -1, 1 \rangle, \langle 3, 0, 2 \rangle \}$

$\vec{R}_1 \quad \vec{R}_2 \quad \vec{R}_3$ $\vec{C}_1 \quad \vec{C}_2$

$$\begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & -1 \\ -1 & 4 & 2 \end{bmatrix} \begin{matrix} C_1 \\ C_2 \end{matrix} \begin{bmatrix} 2 \\ 3 \\ -1 \\ 0 \\ 1 \\ 2 \end{bmatrix} = \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix} \begin{bmatrix} \vec{R}_1 \cdot \vec{C}_1 & \vec{R}_1 \cdot \vec{C}_2 \\ \vec{R}_2 \cdot \vec{C}_1 & \vec{R}_2 \cdot \vec{C}_2 \\ \vec{R}_3 \cdot \vec{C}_1 & \vec{R}_3 \cdot \vec{C}_2 \end{bmatrix}$$

3×3 3×2 3×2

$$= \begin{bmatrix} 1(2) + 2(-1) + 3(1) & 1(3) + 2(0) + 3(2) \\ 3(2) + 2(-1) - 1(1) & 3(3) + 2(0) - 1(2) \\ -1(2) + 4(-1) + 2(1) & -1(3) + 4(0) + 2(2) \end{bmatrix} = \begin{bmatrix} 3 & 9 \\ 3 & 7 \\ -4 & 1 \end{bmatrix}$$

why? one linear eqn in n variables:

$$a_1 x_1 + \dots + a_n x_n = b \quad [a_1 \dots a_n] \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = b$$

more eqns:

$$\begin{aligned} a_{11} x_1 + \dots + a_{1n} x_n &= b_1 \\ a_{21} x_1 + \dots + a_{2n} x_n &= b_2 \end{aligned}$$

$$\begin{bmatrix} a_{11} \dots a_{1n} \\ a_{21} \dots a_{2n} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

collapse vertically

same right factor

in general: $A \vec{x} = \vec{b}$

$m \times n \quad n \times 1 \quad m \times 1$

more variables:

$$\begin{aligned} a_{11} x_1 + \dots + a_{1n} x_n &= b_1 & q_{11} y_1 + \dots + q_{1n} y_n &= c_1 \\ a_{21} x_1 + \dots + a_{2n} x_n &= b_2 & q_{21} y_1 + \dots + q_{2n} y_n &= c_2 \end{aligned}$$

$$\begin{bmatrix} a_{11} \dots a_{1n} \\ a_{21} \dots a_{2n} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \quad \begin{bmatrix} a_{11} \dots a_{1n} \\ a_{21} \dots a_{2n} \end{bmatrix} \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

collapse horizontally

same left factor

$$\begin{bmatrix} a_{11} \dots a_{1n} \\ a_{21} \dots a_{2n} \end{bmatrix} \begin{bmatrix} x_1 & y_1 \\ \vdots & \vdots \\ x_n & y_n \end{bmatrix} = \begin{bmatrix} b_1 & c_1 \\ b_2 & c_2 \end{bmatrix}$$

left is for coefficients, right is for variables, they combine via the linear combination operation to make linear functions (linear homogeneous functions!)

MATRIX MULTIPLICATION FACTOR ORDER MATTERS!
LEFT \neq RIGHT

$$A B \neq B A \quad (\text{in general})$$