

nth order differential equation (DE)

unknown, dependent variable

$$F\left(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \dots, \frac{d^n y}{dx^n}\right) = 0$$

↑
independent variable

if put everything on left hand side of equation, F some expression depending on quantities in parentheses

n = highest derivative appearing in equation

We only consider DEs in which one can explicitly solve for the highest derivative as a function of the remaining quantities:

$$\frac{d^n y}{dx^n} = G\left(x, y, \frac{dy}{dx}, \dots, \frac{d^{n-1} y}{dx^{n-1}}\right) \quad \text{a "standard form"}$$

↓ { given an initial value of x : $x=x_0$ } { pick values of remaining n quantities } { then the highest derivative is determined by the DE }

$$y_0, \frac{dy}{dx}|_{x=x_0}, \dots, \frac{d^{n-1} y}{dx^{n-1}}|_{x=x_0}$$

$$\frac{d^n y}{dx^n}|_{x=x_0} = G\left(x_0, y_0, \frac{dy}{dx}|_{x=x_0}, \dots, \frac{d^{n-1} y}{dx^{n-1}}|_{x=x_0}\right)$$

{ "initial data" for DE at $x=x_0$ }

{ assigning values to these means n conditions for nth order DE }

Example $\frac{d^2y}{dx^2} + 2y = 0 \rightarrow \frac{d^2y}{dx^2} = -2y$

now differentiate both sides:

$$\frac{d^3y}{dx^3} = \frac{d}{dx}(-2y) = -2 \frac{dy}{dx}$$

and again:

$$\frac{d^4y}{dx^4} = \frac{d}{dx}(-2 \frac{dy}{dx}) = -2 \frac{d^2y}{dx^2} = -2(-2y) = 4y$$

etc.

...

in other words, all higher derivatives ($n > 2$) can be determined as functions only of the initial value quantities $y, \frac{dy}{dx}$ for this DE.

Thus given a choice $x=x_0$, setting $y(x_0) = y_0, \frac{dy}{dx}(x_0) = v_0$, then one can evaluate all higher derivatives of y at $x=x_0$ and write a Taylor series representation of the solution function valid within some radius of convergence.

In general specifying n initial conditions for an nth order DE completely determines a unique solution function at least locally. The general solution must therefore contain n arbitrary constants to allow these n initial conditions to be used to solve for values of those constants.