# CHECKING SOLUTIONS OF EQUATIONS

solve an equation for an "unknown" (the variable):

$$x^2 - 3x + 2 = 0 \xrightarrow[\text{technique}]{\text{soln}} \underbrace{\overbrace{x=1, x=2}^{\text{general soln}}}_{\text{a particular soln}}$$

check the solution by substituting it back into the equation & simplifying LHS, RHS independently:

$$(1)^2 - 3(1) + 2 = 0 \qquad (2)^2 - 3(2) + 2 = 0$$
$$1 - 3 + 2 = 0 \qquad 4 - 6 + 2 = 0$$
$$0 = 0 \checkmark \qquad 0 = 0 \checkmark$$

solve a differential equation for an "unknown": the variable function

the solution is an equation expressing the unknown (LHS) as a function of the independent variable (RHS)

1) $\dfrac{dy}{dx} = ky \longrightarrow y = Ce^{kx}$

unknown: $y$; independent variable: $x$; parameter in DE: $k$; "arbitrary constant" (any real number): $C$

$y = 2e^{kx}$ is a particular soln

$y = Ce^{kx}$ is the general soln

check: $\dfrac{d}{dx}(Ce^{kx}) = k(Ce^{kx})$

$C\dfrac{d}{dx}e^{kx} = kCe^{kx}$

$Cke^{kx} = Cke^{kx} \checkmark$

"backsubstitution" everywhere in eqn for unknown

2) $\dfrac{d^2y}{dx^2} + \omega^2 y = 0 \longrightarrow y_1 = C_1 \cos\omega x + C_2 \sin\omega x$ "general soln"

note 2; frequency $\omega > 0$; 2 arbitrary constants

$y = 2\cos\omega x + 4\sin\omega x$ is a particular soln

check:

$$\frac{d^2}{dx^2}(c_1\cos\omega x + c_2\sin\omega x) + \omega^2(c_1\cos\omega x + c_2\sin\omega x) = 0$$

$$\frac{d}{dx}\left(c_1\frac{d}{dx}\cos\omega x + c_2\frac{d}{dx}\sin\omega x\right) + \omega^2 c_1\cos\omega x + \omega^2 c_2\sin\omega x = 0$$

$$\frac{d}{dx}(-c_1\omega\sin\omega x + c_2\omega\cos\omega x) + c_1\omega^2\cos\omega x + c_2\omega^2\sin\omega x = 0$$

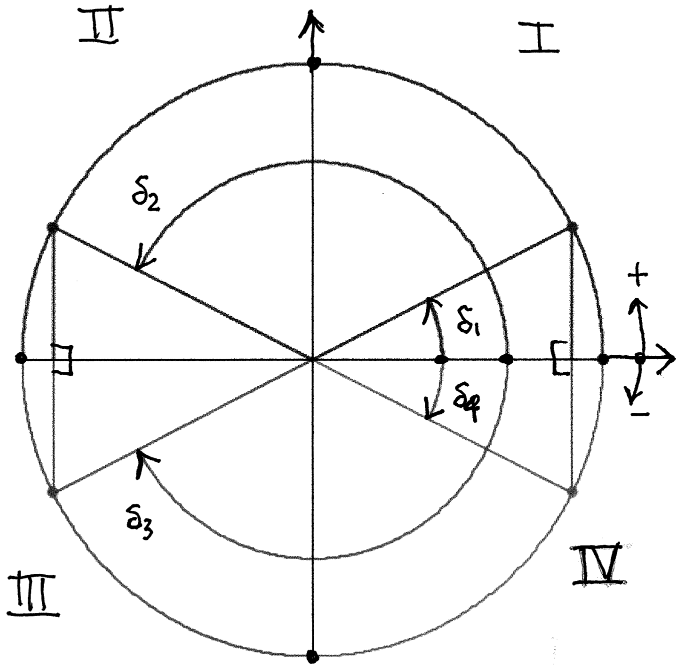
$$-c_1\frac{d}{dx}\sin\omega x + c_2\frac{d}{dx}\cos\omega x + c_1\omega^2\cos\omega x + c_2\omega^2\sin\omega x = 0$$

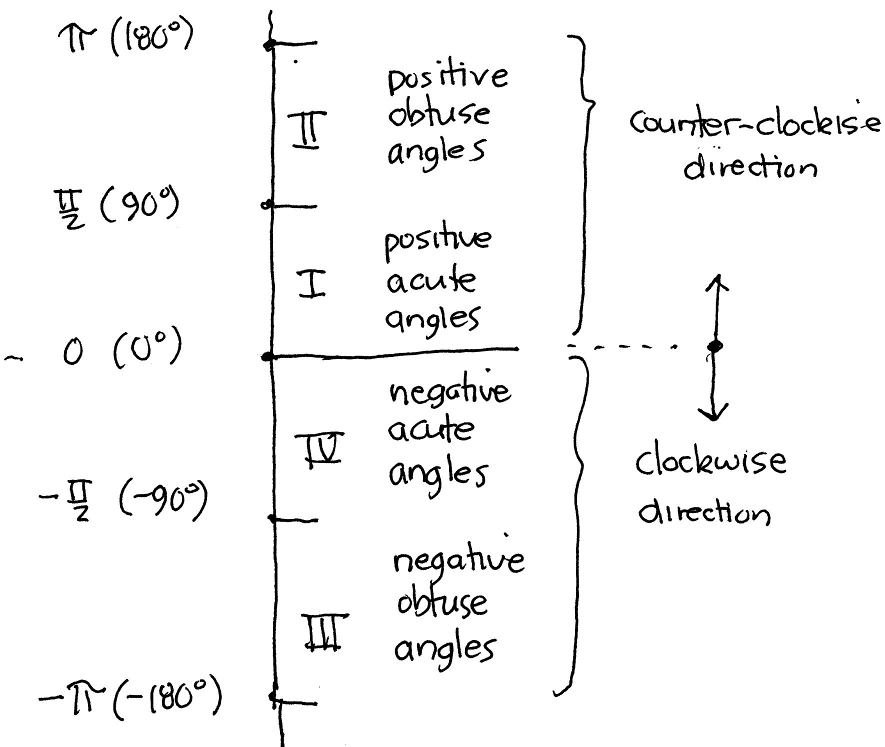
$$\cancel{-c_1\omega^2\cos\omega x} - \cancel{c_2\omega^2\sin\omega x} + \overbrace{\cancel{c_1\omega^2\cos\omega x}}^{0} + \overbrace{\cancel{c_2\omega^2\sin\omega x}}^{0} = 0$$

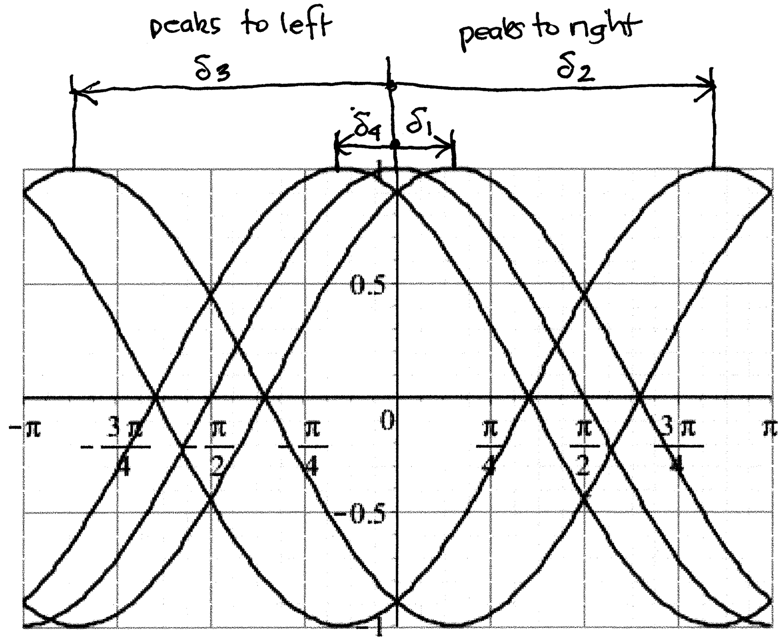
$$0 = 0 \checkmark$$

memorize these 2 DE solutions. we will derive their solutions later but will use them throughout the semester. exponentials describe growth ($k>0$) and decay ($k<0$) while cosines & sines describe oscillations. (of frequency $\omega>0$).

Choosing an angle $-\pi \le \delta \le \pi$ for all directions on a circle (divided into 4 quadrants)

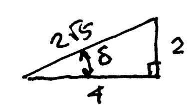



$\cos(x-\delta) = \cos\delta \cos x + \sin\delta \sin x$

"reference" triangle example




$\tan\delta = 2/4 = 1/2$

$\delta = \arctan(1/2)$

"translating" the cosine graph leads to a combination of cosine & sine graphs with a peak "shifted" to the right ($\delta > 0$) or left ($\delta < 0$) compared to the standard cosine graph

$\delta$ = "delta" = phase shift

$|\delta| \le \pi$ !