figure8curve.mw resonance calculation (particular soln is the response to driving force)

$$\begin{bmatrix} x_1'' \\ x_2'' \end{bmatrix} = \underbrace{\begin{bmatrix} -5/2 & 3/2 \\ 3/2 & -5/2 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 50\cos\omega t \end{bmatrix}$$

$$A \to B = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \quad B^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \quad B^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} \quad A_B = \begin{bmatrix} -1 & 0 \\ 0 & -4 \end{bmatrix}$$

$\lambda = \; -1 \; -4$

$$\begin{bmatrix} y_1'' \\ y_2'' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -4 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + 25\cos\omega t \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -y_1 + 25\cos\omega t \\ -4y_2 + 25\cos\omega t \end{bmatrix}$$

$y_1'' + y_1 = 25\cos\omega t$

$y_{1p} = C_5\cos\omega t + C_6\sin\omega t$

$y_{1p}'' = -\omega^2 C_5\cos\omega t - \omega^2 C_6\sin\omega t$

$y_{1p}'' + y_{1p} = \underbrace{(1-\omega^2)C_5}_{=25 \to C_5 = \frac{25}{1-\omega^2}}\cos\omega t + \underbrace{(1-\omega^2)C_6}_{=0 \to C_6 = 0}\sin\omega t = 25\cos\omega t$

$= \boxed{\frac{25}{1-\omega^2}\cos\omega t = y_{1p}}$

$y_2'' + 4y_2 = 25\cos\omega t$

$y_{2p} = C_7\cos\omega t + C_8\sin\omega t$

$y_{2p}'' = -\omega^2 C_7\cos\omega t - \omega^2 C_8\sin\omega t$

$y_{2p}'' + 4y_{2p} = \underbrace{(4-\omega^2)C_7}_{=25 \to C_7 = \frac{25}{4-\omega^2}}\cos\omega t + \underbrace{(4-\omega^2)C_8}_{=0 \to C_8 = 0}\sin\omega t = 25\cos\omega t.$

$= \boxed{\frac{25}{4-\omega^2}\cos\omega t = y_{2p}}$

$$\underbrace{\vec{x}_p}_{\cos\omega t} = \underbrace{B\vec{y}_p}_{\cos\omega t} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 25/(1-\omega^2) \\ 25/(4-\omega^2) \end{bmatrix} = 25\begin{bmatrix} \frac{1}{1-\omega^2} - \frac{1}{4-\omega^2} \\ \frac{1}{1-\omega^2} + \frac{1}{4-\omega^2} \end{bmatrix} = 25\frac{\begin{bmatrix} (4-\omega^2)-(1-\omega^2) \\ 4-\omega^2+(1-\omega^2) \end{bmatrix}}{(1-\omega^2)(4-\omega^2)} = 25\frac{\begin{bmatrix} 3 \\ 5-2\omega^2 \end{bmatrix}}{(1-\omega^2)(4-\omega^2)}$$

$$\boxed{\vec{x}_p = \begin{bmatrix} x_{1p} \\ x_{2p} \end{bmatrix} = \frac{25\cos\omega t}{(1-\omega^2)(4-\omega^2)}\begin{bmatrix} 3 \\ 5-2\omega^2 \end{bmatrix}} = \begin{bmatrix} a(\omega)\cos\omega t \\ b(\omega)\cos\omega t \end{bmatrix}$$

$5-2\omega^2 = 0 \to \omega^2 = 5/2$

$\omega_{crit} = \sqrt{5/2} \approx 1.58 \quad x_2 = 0$ mass 2 remains at rest at equilibrium (mass 1 always oscillates, pushing on mass 2 to oppose F_2)

$0 < \omega < \omega_{crit}$: tandem mode excited $0<\omega<1$ or $\omega>2$ inphase with force

$\omega > \omega_{crit}$: accordian mode excited $1<\omega<2$ 180° out of phase with force

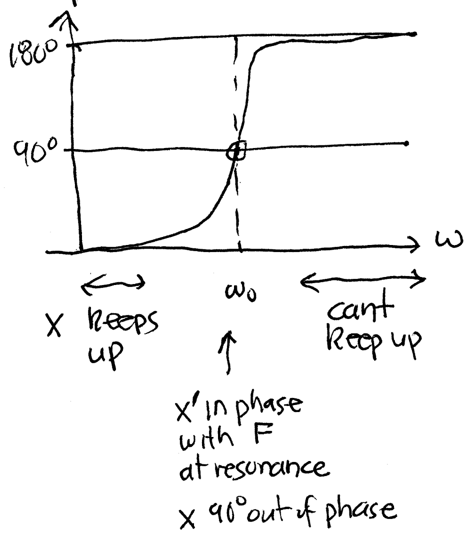
Apart from slightly different numbers problem E&P3 7.4.3,9 behaves exactly this way.

figure8curve.mw understanding phases of response variables done

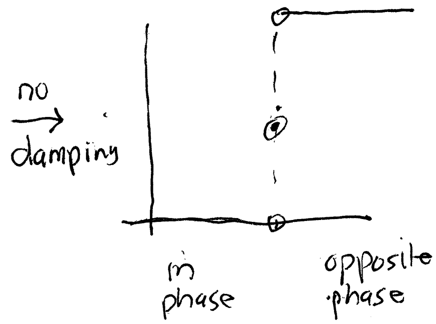
You wont find this in any book but it shows how we can draw general conclusions about the behavior of physical systems rather than just being able to write down some formal solution of a DE system.

The power of mathematics is that it can lead to UNDERSTANDING.
It is not just a collection of recipes to get answers.

phase shift for a single oscillator with slight damping: $x = A(\omega)\cos\omega t$, $F = B_0\cos\omega t$



no damping →

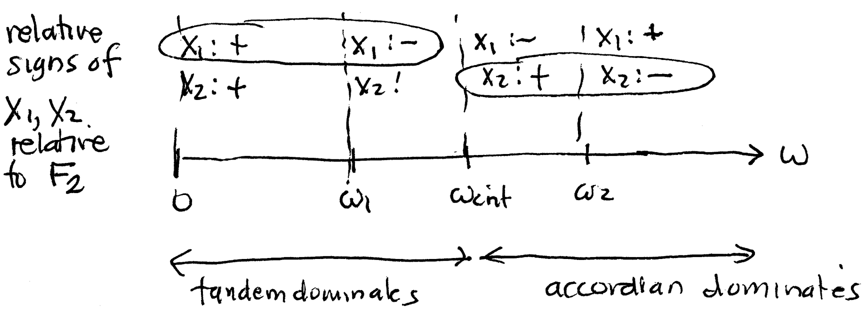


$$x = \frac{B_0}{\omega_0^2-\omega^2}\cos\omega t$$

> 0 if $\omega < \omega_0$
$= 0$ if $\omega = \omega_0$
< 0 if $\omega > \omega_0$

Passing through resonance where the response amplitude gets very large, the phase shift of the position x relative to the force F goes from 0 to 180°. When slow compared to resonance frequency ~~x~~ keeps up with F, when fast compared to that frequency, x cant keep up and falls behind so always out of phase.

two oscillators (no damping)



x_1 goes through expected phase transition
x_2 along for ride.

x_2 goes through expected phase transition
x_1 along for the ride

you can safely ignore this discussion if it does not appeal to you.