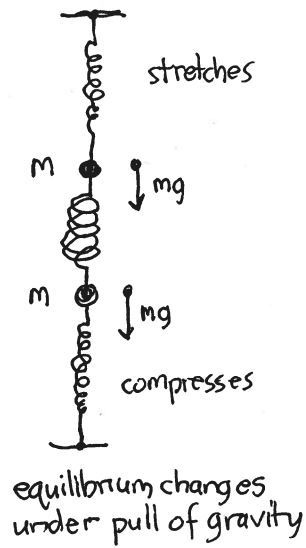


2 equal mass 3 symmetrically arranged springs with gravity: equilibrium



$$m_1 x_1'' = -k_1 x_1 + k_2 (x_2 - x_1) + m_1 g$$

$$m_2 x_2'' = -k_2 (x_2 - x_1) - k_3 x_2 + m_2 g$$

equal masses: $m_1 = m_2 = m$
 equal outersprings: $k_1 = k_3 = k$
 50% stiffer middle spring: $k_2 = \frac{3}{2}k$

$$m x_1'' = -k x_1 + \frac{3}{2}k (x_2 - x_1) + mg = 0$$

$$m x_2'' = -\frac{3}{2}k (x_2 - x_1) - k x_2 + mg = 0$$

$$-k x_{10} + mg = 0$$

$$-k x_{20} + mg = 0$$

$$x_1'' = \frac{k}{m} \left[-\frac{5}{2} x_1 + \frac{3}{2} x_2 \right] + g = 0$$

$$x_2'' = \frac{k}{m} \left[\frac{3}{2} x_1 - \frac{5}{2} x_2 \right] + g = 0$$

equilibrium: $\vec{x} = \vec{x}_0$:

$$\frac{1}{2} \begin{bmatrix} -5 & 3 \\ 3 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + g \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 0$$

Inverse

$$2 \begin{pmatrix} 1 \\ 25-16 \end{pmatrix} \begin{bmatrix} -5 & 3 \\ -3 & -5 \end{bmatrix} = -\frac{1}{8} \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}$$

$$\begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix} \cdot \frac{g}{k/m} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{g}{k/m} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

equilibrium.

Since $x_{10} = x_{20}$, then $x_{20} - x_{10} = 0$ & RHS simplifies to

$x_{10} = x_{20}$ so middle spring is unstretched
 upperspring stretched by same amount
 lower spring is compressed
 namely by weight of one mass

The force free equations describe oscillations about this new equilibrium.

optional exercise

- What is the new equilibrium position vector \vec{x}_0 for general values of m_i and k_i ?
- If only $m_1 = m_2$, does the middle spring stretch or contract? Does this make "common sense"?
- How do things change if $k_3 = 0$, i.e., we remove the ~~third~~ spring?
- How do the eigenfrequencies change if we remove the third spring?
 would you expect the motion to slow down or speed up if you remove one of the forces?
- Can you think of other simple questions we could ask about this system?