

## figure8curve.mw

driven case

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}'' = \underbrace{\begin{bmatrix} -5/2 & 3/2 \\ 3/2 & -5/2 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 50 \cos 3t \end{bmatrix} + 50 \cos 3t$$

decoupling  
soln:

$$\lambda = -1 - 4$$

$$B = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$B^{-1} = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$AB = B^{-1}AB = \begin{bmatrix} -1 & 0 \\ 0 & -4 \end{bmatrix}$$

$$B^{-1}\vec{F} = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 50 \cos 3t \end{bmatrix} = \begin{bmatrix} 25 \cos 3t \\ 25 \cos 3t \end{bmatrix}$$

$$\vec{x} = B\vec{y} = y_1 \vec{b}_1 + y_2 \vec{b}_2$$

$$\vec{x}'' = A\vec{x} + \vec{F} \Rightarrow \vec{y}'' = AB\vec{y} + B^{-1}\vec{F} : \begin{bmatrix} y_1'' \\ y_2'' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} 25 \cos 3t \\ 25 \cos 3t \end{bmatrix} = \begin{bmatrix} -y_1 + 25 \cos 3t \\ -4y_2 + 25 \cos 3t \end{bmatrix}$$

$$y_1'' + y_1 = 25 \cos 3t$$

$$y_{1h} = C_1 \cos 3t + C_2 \sin 3t$$

$$y_{1p} = C_5 \cos 3t + C_6 \sin 3t$$

$$y_2'' + 4y_2 = 25 \cos 3t$$

$$y_{2h} = C_3 \cos 3t + C_4 \sin 3t$$

$$y_{2p} = C_7 \cos 3t + C_8 \sin 3t$$

$$y_{1p}'' + y_{1p} = -9C_5 \cos 3t - 9C_6 \sin 3t + C_5 \cos 3t + C_6 \sin 3t = \frac{-8C_5 \cos 3t}{=25} - \frac{-8C_6 \sin 3t}{=0} = 25 \cos 3t$$

$$C_5 = -\frac{25}{8} \quad C_6 = 0.$$

$$y_{2p}'' + 4y_{2p} = -9C_7 \cos 3t - 9C_8 \sin 3t + 4C_7 \cos 3t + 4C_8 \sin 3t = \frac{-5C_7 \cos 3t}{=25} - \frac{5C_8 \sin 3t}{=0} = 25 \cos 3t$$

$$C_7 = -\frac{25}{5} = -5 \quad C_8 = 0$$

initial values

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} C_1 \cos t + C_2 \sin t \\ C_3 \cos 2t + C_4 \sin 2t \end{bmatrix} - \frac{25}{8} \cos 3t \quad \left. \begin{array}{l} \text{general} \\ \text{soln} \end{array} \right\}$$

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -C_1 \sin t + C_2 \cos t \\ -2C_3 \sin 2t + 2C_4 \cos 2t \end{bmatrix} + \frac{75}{8} \sin 3t$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} C_1 - 25/8 \\ C_3 - 5 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/2 \\ -1/2 \end{bmatrix} \quad C_1 = 1/2 + 25/8 = 29/8$$

$$C_3 = 5 - 1/2 = 9/2$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x_1'(0) \\ x_2'(0) \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} C_2 \\ 2C_4 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} \rightarrow \quad C_2 = 1/2$$

$$C_4 = 1/4.$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{29}{8} \cos t + \frac{1}{2} \sin t - \frac{25}{8} \cos 3t \\ \frac{9}{2} \cos 2t + \frac{1}{4} \sin 2t - 5 \cos 3t \end{bmatrix}$$

← matrix form

$$= \begin{bmatrix} \frac{29}{8} \cos t + \frac{1}{2} \sin t & -\frac{25}{8} \cos 3t \\ \frac{29}{8} \cos t + \frac{1}{2} \sin t & -\frac{9}{2} \cos 2t - \frac{1}{4} \sin 2t + 5 \cos 3t \end{bmatrix}$$

← scalar form  
(top, bottom)

$$= \left( \frac{29}{8} \cos t + \frac{1}{2} \sin t \right) \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \left( \frac{9}{2} \cos 2t + \frac{1}{4} \sin 2t \right) \begin{bmatrix} -1 \\ 1 \end{bmatrix} + \frac{5}{8} \cos 3t \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

← vector linear comb form

$\omega = 1$  natural mode

$\omega = 2$  natural mode

$\omega = 3$  response mode

$\cos(t - \delta)$

$$\delta_1 = \arctan\left(\frac{4}{29}\right) \approx 3.9^\circ$$

$$A_1 = \frac{1}{2} \sqrt{29^2 + 4^2} \approx 3.66$$

$$\delta_2 = \arctan\left(\frac{1}{18}\right) \approx 0.009^\circ$$

$$A_2 = \frac{1}{2} \sqrt{18^2 + 1^2} \approx 4.51$$

(see Maple worksheet graphic)