

$$\vec{x}'' = A\vec{x} \quad \text{IVP (2x2 coefficient matrix)}$$

$$\begin{aligned} x_1'' &= (-5x_1 + 3x_2)/2, \quad x_1(0) = 1, \quad x_1'(0) = 0 \\ x_2'' &= (3x_1 - 5x_2)/2, \quad x_2(0) = 0, \quad x_2'(0) = 1 \end{aligned} \quad \left. \right\} \quad \text{IVP}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}'' = \frac{1}{2} \begin{bmatrix} -5 & 3 \\ 3 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} x_1'(0) \\ x_2'(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \text{matrix form}$$

$$0 = \det(A - \lambda I) = \lambda^2 + 5\lambda + 4 = (\lambda + 1)(\lambda + 4) \quad \left. \right\} \quad \text{diagonalization, new variables}$$

$$\lambda = -1, -4 \rightarrow \dots \quad B = \langle \vec{b}_1, \vec{b}_2 \rangle = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$B^{-1} = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \quad A_B = B^{-1}AB = \begin{bmatrix} -1 & 0 \\ 0 & 4 \end{bmatrix}$$

$$\vec{x} = B\vec{y} = y_1 \vec{b}_1 + y_2 \vec{b}_2, \quad \vec{y} = B^{-1}\vec{x}$$

$$\vec{x}'' = A\vec{x} \rightarrow B^{-1}(B\vec{y})'' = BA B\vec{y} \quad \left. \right\} \quad \text{decoupling DEs}$$

$$\vec{y}'' = A_B \vec{y}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}'' = \begin{bmatrix} -1 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} -y_1 \\ 4y_2 \end{bmatrix}$$

$$\begin{aligned} y_1'' &= -y_1 \quad y_1 = c_1 \cos t + c_2 \sin t \\ y_2'' &= 4y_2 \quad y_2 = c_3 \cos 2t + c_4 \sin 2t \end{aligned}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = B \begin{bmatrix} c_1 \cos t + c_2 \sin t \\ c_3 \cos 2t + c_4 \sin 2t \end{bmatrix} \quad \downarrow \text{solving ICs}$$

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = B \begin{bmatrix} -c_1 \sin t + c_2 \cos t \\ -2c_3 \sin 2t + 2c_4 \cos 2t \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = B \begin{bmatrix} c_1 \\ c_3 \end{bmatrix}, \quad \begin{bmatrix} c_1 \\ c_3 \end{bmatrix} = B^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x_1'(0) \\ x_2'(0) \end{bmatrix} = B \begin{bmatrix} c_2 \\ 2c_4 \end{bmatrix}, \quad \begin{bmatrix} c_2 \\ 2c_4 \end{bmatrix} = B^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} c_1 \\ c_3 \end{bmatrix} = \begin{bmatrix} 1/2 \\ -1/2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(\cos t + \sin t) \\ \frac{1}{2}(-2\cos 2t + \sin 2t) \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\cos(t - \pi/4) \\ \frac{\sqrt{5}}{4}\cos(2t - (\pi - \arctan \frac{1}{2})) \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} (\cos t + \sin t)/2 \\ (-2\cos 2t + \sin 2t)/4 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2}\cos t + \frac{1}{2}\sin t + \frac{1}{2}\cos 2t - \frac{1}{4}\sin 2t \\ \frac{1}{2}\cos t - \frac{1}{2}\sin t - \frac{1}{2}\cos 2t + \frac{1}{4}\sin 2t \end{bmatrix}$$

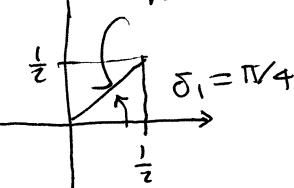
$$\vec{x} = \frac{1}{2}\cos(t - \delta_1) \vec{b}_1 + \frac{\sqrt{5}}{4}\cos(2t - \delta_2) \vec{b}_2$$

slow mode

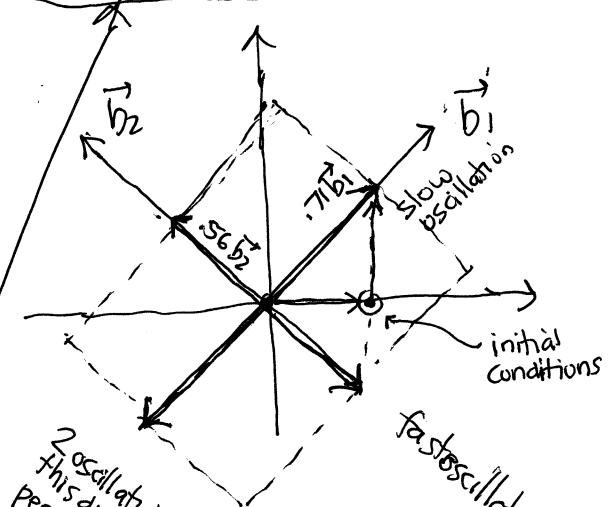
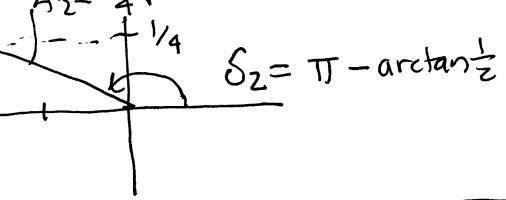
fast mode

phase-shifted cosine calculation

$$A_1 = \frac{1}{2}\sqrt{2} = \frac{1}{\sqrt{2}} \approx .71$$



$$A_2 = \frac{1}{4}\sqrt{4+1} = \sqrt{5}/4 \approx .56$$



This direction
per 1 oscillation
in the other
so we have to get a figure 8 like curve

figure 8 like curve