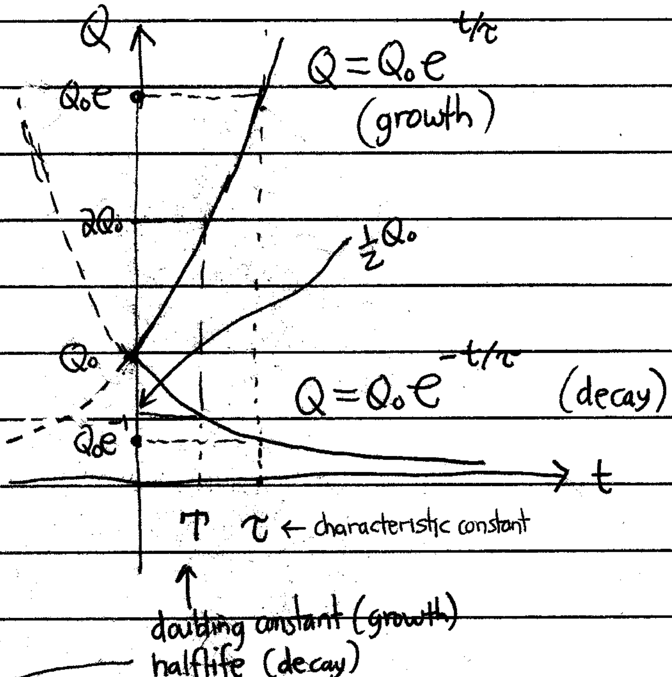
exponential behavior (growth / decay)

$$\operatorname{sgn} k = \begin{cases} + & \text{growth} \\ - & \text{decay} \end{cases}$$

$$Q = Q_0 e^{kt} = Q_0 e^{\pm t/\tau} \quad \text{where} \quad k = \pm \underbrace{|k|}_{= 1/\tau > 0} \quad \text{or } \tau = 1/|k|.$$



τ is the "characteristic constant"
(characteristic time if t is a time variable
characteristic length if t is a length variable)

For any interval of time equal to the characteristic time, the quantity increases by a factor of $e \approx 3$ in the growth case and decreases by a factor of $e^{-1} \approx 1/3$ in the decay case:

$$\begin{cases} Q(t+\tau) = Q_0 e^{\pm(t+\tau)/\tau} = Q_0 e^{\pm t/\tau} e^{\pm 1} \\ Q(t) = Q_0 e^{\pm t/\tau} \end{cases}$$

$$\text{so } \frac{Q(t+\tau)}{Q(t)} = e^{\pm 1} = \begin{cases} e & \text{growth} \\ 1/e & \text{decay} \end{cases}$$

τ is the time scale over which something interesting happens. For example, if you look on the millisecond scale of an oscilloscope at a signal decaying with a time constant of 1 second, it will look constant, but if the time constant is a microsecond the signal will be essentially zero over the millisecond duration of time since it will have quickly decayed away.

continuously compounded interest: $\underbrace{Q}_{\text{principal plus interest}} = \underbrace{Q_0}_{\text{principal}} e^{rt}$ (r ← fractional interest rate; t ← time in years)

$r = .03 \Leftrightarrow 3\% \rightarrow \tau = \frac{1}{r} = \frac{1}{.03} \approx 33$ (years) $\approx$ half a lifetime!

$r = .10 \Leftrightarrow 10\% \rightarrow \tau = \frac{1}{r} = \frac{1}{.10} = 10$ (years) $\approx$ reasonable wait (?)

Relation to halflife T: $\frac{1}{2}\cancel{Q_0} = \cancel{Q_0} e^{-T/\tau} \rightarrow 2 = e^{T/\tau} \rightarrow \ln 2 = \frac{T}{\tau} \rightarrow T = (\ln 2)\tau \approx (.7)\tau$

or doubling time T: $2\cancel{Q_0} = \cancel{Q_0} e^{T/\tau} \rightarrow \ln 2 = \frac{T}{\tau} \quad T = (\ln 2)\tau \approx .7\tau$