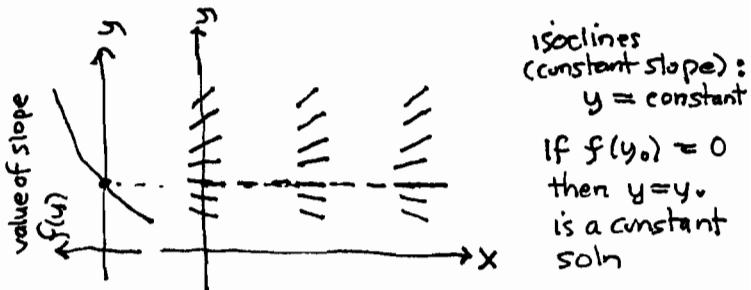
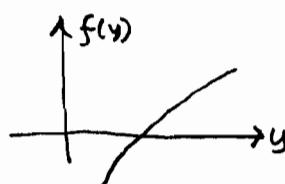


## separable DEs that don't involve the independent variable

$$\frac{dy}{dx} = f(y) \rightarrow \int \frac{dy}{f(y)} = \int dx = x + c_1 \rightarrow G(y) = x + c_1 \rightarrow y = G^{-1}(x + c_1)$$

no x-dependence       $\underbrace{\int \frac{dy}{f(y)}}_{G(y)}$  antiderivative      implicit soln      try to solve for y

direction field does not depend on x:



Simplest cases for  $f(y)$ :

power function

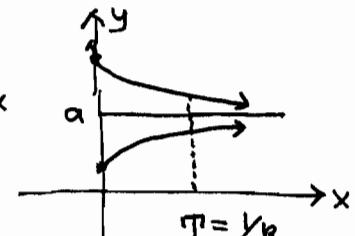
$$\frac{dy}{dx} = k(y-a)^p, p \neq 1 \quad \int (y-a)^{1-p} dy = \int k dx = kx + c_1 \quad \text{solve } y-a = [(kx+c_1)]^{\frac{1}{1-p}}$$

$$\int (y-a)^{-p} dy = \int k dx = kx + c_1$$

linear function (Newton's law of heating/cooling)

$$\frac{dy}{dx} = -k(y-a) \quad \int \frac{dy}{y-a} = -\int k dx = -kx + c_1 \xrightarrow{\text{exp}} |y-a| = c_2 e^{-kx}$$

If  $k > 0$ , difference decays away  
characteristic "time"/"length" tells how long for difference to decrease by factor of  $e$ .



quadratic function (logistic D.E.) {population growth with max stable population)  
(market saturation)}

$$\frac{dy}{dx} = -k(y-a)(y-b), a < b$$

$$\int \frac{dy}{(y-a)(y-b)} = -\int k dx$$

$$\frac{1}{b-a} \ln \left| \frac{y-b}{y-a} \right| = -kx + c_1$$

$$\left| \frac{y-b}{y-a} \right| = c_2 e^{-k(b-a)x} \xrightarrow{\text{solve for } y} y = \dots$$

If  $k > 0$ , this difference ratio decays away on a time/length scale  $T = 1/k$

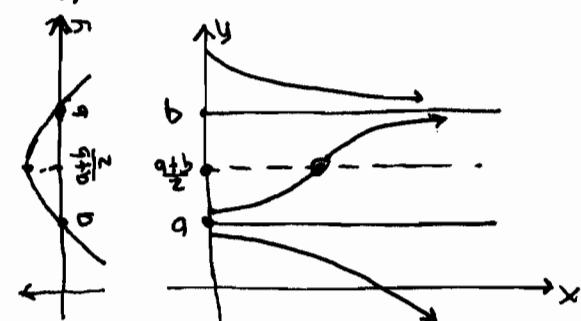
example:  $a=0, b=1$  &  $0 < y < 1$ :  $\rightarrow \begin{cases} K=k \\ T=1/k \end{cases}$

$$\frac{1-y}{y} = c e^{-kx} \rightarrow \frac{1}{y} = 1 + c e^{-kx}$$

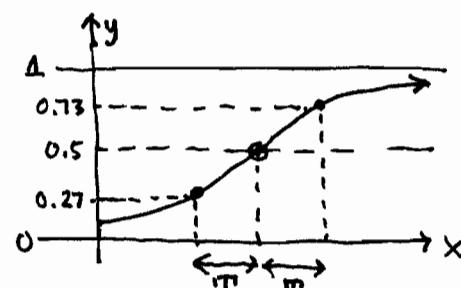
$$\frac{1}{y-1} \quad y = \frac{1}{1+c e^{-kx}}$$

$x=0$ :  $\frac{1-y_0}{y_0} = C$  measures how far from center; if  $y_0 = \frac{1}{2} \rightarrow C=1$

$$x=T: \frac{1-y}{y} = C e^{-1} = e^{-1} \rightarrow \frac{1}{y-1} = e^{-1} \rightarrow y = \frac{1}{1+e^{-1}} \approx 0.731$$



"S-curve" between a and b with max slope midway (pt of inflection)



In an interval of  $2T$ , y travels about half distance between 0 and 1.