definite integrals and the IVP solution

$$\frac{dy}{dx} = f(x)\ , \quad y(a) = y_0$$

This is the next to the last step in the solution of a first order linear DE for product of unknown and the integrating factor

$$y = \underbrace{\int f(x)\,dx}_{F(x)\ \text{antiderivative}} + C$$

if $F(a) = 0$, then $C = y(a) = y_0$ is the IVP solution constant

so $\quad y = F(x) + y(a)$ is the IVP soln.

Define $\quad \int_a^x f(t)\,dt = F(x)$, then $\begin{cases} F'(x) = f(x) & \text{(Fund. Theorem of Calc.)} \\ F(a) = \int_a^a f(t)\,dt = 0 \end{cases}$

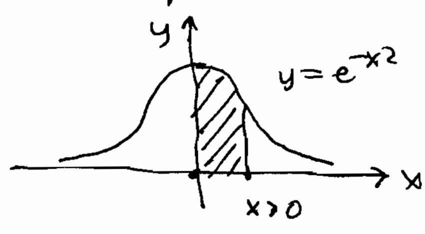
Thus $\quad y(x) = \int_a^x f(x)\,dx + y(a) \quad$ is the IVP soln

When f does not have an antiderivative that can be expressed in terms of elementary functions, this formula provides an expression for a convenient antiderivative that we can work with, and can be used to define a new special function.

Ex. $\quad \dfrac{dy}{dx} = e^{-x^2}\ , \quad y(0) = 0$

$$y = \int_0^x e^{-t^2}\,dt$$

"bell shaped curve"



area from 0 to x under curve.

Apart from a multiplicative constant, this defines the error function erf(x) used as a probability distribution (and closely related to the normal distribution used in curving grades!)

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\,dt \quad \rightarrow \mathrm{erf}(\infty) = 1$$

The multiplicative constant makes the total area under the curve for $x \geq 0$ equal to 1 (necessary for a probability distribution)

(see WikiPedia)