ECP3 2.3.22. $v \le 0$ case air resistance $(v \gg v_T)$

$$\underbrace{\frac{dv}{dt}}_{\frac{v_T\, d(v/v_T)}{dt}} = -g + \rho v^2 = -g\left(1 - \frac{\rho}{g} v^2\right) = -g\left(1 - \underbrace{\left(\frac{v}{v_T}\right)^2}_{u^2}\right), \quad v_T = \sqrt{\frac{g}{\rho}}, \quad u = \frac{v}{v_T}$$

$$\underbrace{\int \frac{du}{1-u^2}}_{\tanh^{-1} u \text{ if } |u|<1} = \int -\frac{g}{v_T}\, dt = -\sqrt{\rho g}\, t + C_1 \rightarrow u = \tanh(C_1 - \sqrt{\rho g}\, t)$$

$$\boxed{v = v_T \tanh(\underbrace{C_1 - \sqrt{\rho g}\, t}_{\text{must be dimensionless}})}$$

Given: $g = 32\ ft/s^2$, $\rho = 0.075$ $\Big\}$ $v_T = \sqrt{\frac{32}{0.075}} = 20.6559 \approx 20.7\ ft/s^2$

$\sqrt{\rho g} = 1.54919\ s^{-1}$

$\tau = \frac{1}{\sqrt{\rho g}} = 0.645497\, s$

since $\sqrt{\rho g}$ must have inverse time units, its reciprocal must have time units — this defines the characteristic time!

I.C. $v(0) = 0$ at jump, $y(0) = 10{,}000\ ft.$

$0 = v_T \tanh C_1 \rightarrow C_1 = 0$

$v = v_T \tanh(-\sqrt{\rho g}\, t) = -v_T \tanh(\sqrt{\rho g}\, t)$

$y = \int v\, dt = -v_T \int \tanh \sqrt{\rho g}\, t\, dt \quad \leftarrow \quad \int \tanh(ax)\, dx = \frac{1}{a} \ln \cosh(ax) + C$

$= \frac{-v_T}{\sqrt{\rho g}} \ln \cosh(\sqrt{\rho g}\, t) + C$

$10{,}000 = y(0) = -\frac{v_T}{\sqrt{\rho g}} \underbrace{\ln \underbrace{\cosh(0)}_{1}}_{0} + C \rightarrow C = 10{,}000$

$$\boxed{y = -\frac{v_T}{\sqrt{\rho g}} \ln \cosh(\sqrt{\rho g}\, t) + 10000}$$

$\frac{v_T}{\sqrt{\rho g}} \doteq 13.3333\, ft = \underbrace{v_T \tau}_{}$

distance traveled at terminal velocity for characteristic time interval (sets scale for change in y)

number of multiples of this distance in initial height 10,000 ft: 750

Hits ground at $y = 0$, solve for t: (use $\sqrt{\rho g} = \frac{1}{\tau}$)

$0 = -v_T \tau \ln \cosh(t/\tau) + 10000$

$\ln \cosh(t/\tau) = \frac{10000}{v_T \tau}$

$\cosh(t/\tau) = e^{10000/(v_T \tau)}$

$\frac{t}{\tau} = \cosh^{-1} e^{10000/(v_T \tau)} \longrightarrow t = \tau \cosh^{-1} e^{\overbrace{10{,}000/(v_T \tau)}}$

sets scale of result!

numerical factor ≈ 750.255 — nearly the same!

$\approx 484.570 \approx 480 + 5 = \boxed{8\text{ min } 5\text{ s}}$