

E8P 7.4: 3,9 2 mass-3 spring system initially at rest, no velocity

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = A \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \underbrace{\frac{B_0}{M_2} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cos \omega t}_{\vec{F}}$$

$$A = \begin{bmatrix} -\frac{(k_1+k_2)}{m_1} & \frac{k_2}{m_1} \\ \frac{k_2}{m_2} & -\frac{(k_2+k_3)}{m_2} \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 1 & -2 \end{bmatrix} \text{ for } m_1=1, m_2=2 \\ k_1=1, k_2=2=k_3 \\ B_0=120, \omega=3$$

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + 60 \cos 3t \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$A \rightarrow \lambda = -1, -4$$

$$B = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix} B^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$$

$$A_D = B^{-1}AB = \begin{bmatrix} -1 & 0 \\ 0 & 4 \end{bmatrix}$$

$$\begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} + \begin{bmatrix} 40 \cos 3t \\ 20 \cos 3t \end{bmatrix}$$

$$y_1'' = -y_1 + 40 \cos 3t$$

$$y_2'' = -4y_2 + 20 \cos 3t$$

$$y_1'' + y_1 = 40 \cos 3t$$

$$y_2'' + 4y_2 = 20 \cos 3t$$

$$B^{-1}\vec{F} = \frac{1}{3} \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} 60 \cos 3t = 20 \cos 3t \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\vec{x} = B \vec{y} = y_1 \vec{b}_1 + y_2 \vec{b}_2, \quad \vec{y} = B^{-1} \vec{x}$$

$$\vec{x}'' = A \vec{x} + \vec{F} \rightarrow \vec{y}'' = A_D \vec{y} + B^{-1} \vec{F}$$

decoupled equations.

$$y_{1h} = C_1 \cos 3t + C_2 \sin 3t \quad y_{1p} = C_5 \cos 3t + C_6 \sin 3t$$

$$y_{2h} = C_3 \cos 2t + C_4 \sin 2t \quad y_{2p} = C_7 \cos 3t + C_8 \sin 3t$$

$$1 [y_{1p} = C_5 \cos 3t + C_6 \sin 3t]$$

$$0 [y_{1p}' = -3C_5 \sin 3t + 3C_6 \cos 3t]$$

$$1 [y_{2p}'' = -9C_7 \cos 3t - 9C_8 \sin 3t]$$

$$4 [y_{2p} = C_7 \cos 3t + C_8 \sin 3t]$$

$$0 [y_{2p}' = -3C_7 \sin 3t + 3C_8 \cos 3t]$$

$$1 [y_{2p}'' = -9C_7 \cos 3t - 9C_8 \sin 3t]$$

$$y_{1p}'' + y_{1p} = \underbrace{(-9)C_5 \cos 3t}_{= 40} + \underbrace{(-9)C_6 \sin 3t}_{= 0} = 40 \cos 3t \rightarrow C_5 = \frac{-40}{9} = -5$$

$$y_{1p} = -5 \cos 3t$$

$$y_1 = c_1 \cos 3t + c_2 \sin 3t - 5 \cos 3t$$

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \cos 3t + c_2 \sin 3t - 5 \cos 3t \\ c_3 \cos 2t + c_4 \sin 2t - 4 \cos 3t \end{bmatrix}$$

$$\begin{bmatrix} \ddot{x}_1' \\ \ddot{x}_2' \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -c_1 \sin 3t + c_2 \cos 3t + 15 \sin 3t \\ -2c_3 \sin 2t + 2c_4 \cos 2t + 12 \sin 3t \end{bmatrix}$$

$$y_1 = 5 \cos t - 5 \cos 3t, y_2 = 4 \cos 2t - 4 \cos 3t \quad \leftarrow \rightarrow c_1 = 5, c_3 = 4, c_2 = 0, c_4 = 0$$

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 5 \cos t - 5 \cos 3t \\ 4 \cos 2t - 4 \cos 3t \end{bmatrix} = \begin{bmatrix} 5 \cos t - 5 \cos 3t - 2(4 \cos 2t - 4 \cos 3t) \\ 5 \cos t - 5 \cos 3t + (4 \cos 2t - 4 \cos 3t) \end{bmatrix} = \begin{bmatrix} 5 \cos t - 8 \cos 2t + 3 \cos 3t \\ 5 \cos t + 4 \cos 2t - 9 \cos 3t \end{bmatrix}$$

$$= \cos t \begin{bmatrix} 5 \\ 6 \end{bmatrix} + \cos 2t \begin{bmatrix} -8 \\ 4 \end{bmatrix} + \cos 3t \begin{bmatrix} 3 \\ -9 \end{bmatrix}$$

$$\omega_1 = 1 \text{ tandem} \quad T_1 = 2\pi$$

$$\omega_2 = 2 \text{ accordion} \quad T_2 = \pi$$

$$1:1 \text{ amp ratio}$$

$$2:1 \text{ amp ratio}$$

$$\omega_3 = 3 \text{ accordion} \quad T_3 = 2\pi/3$$

$$1:3 \text{ amp ratio}$$

$$\text{common period: } 2\pi = T_1 = 2T_2 = 3T_3$$

3 independent modes oscillate back and forth between tips of arrows. The combination oscillates from the origin along the curve to the point $(-16, 0)$ and back

The 3 vectors sum to zero to satisfy $\vec{x}(0) = \vec{0}$.

