E&P 7.4,3,9 resonance calculation: 3→ω

$$\begin{bmatrix} x_1'' \\ x_2'' \end{bmatrix} = \underbrace{\begin{bmatrix} -3 & 2 \\ 1 & -2 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \overbrace{60 \cos \omega t \begin{bmatrix} 0 \\ 1 \end{bmatrix}}^{\vec f}$$

$A \rightarrow$ $\lambda = -1, -4$

$$B = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix} \rightarrow B^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} \rightarrow B^{-1}\vec f = 20 \cos \omega t \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= 20 \cos \omega t \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 40 \cos \omega t \\ 20 \cos \omega t \end{bmatrix}$$

$$A_B = \begin{bmatrix} -1 & 0 \\ 0 & -4 \end{bmatrix}$$

$$\begin{bmatrix} y_1'' \\ y_2'' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -4 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} 40 \cos \omega t \\ 20 \cos \omega t \end{bmatrix} = \begin{bmatrix} -y_1 + 40 \cos \omega t \\ -4y_2 + 20 \cos \omega t \end{bmatrix}$$

decoupled eqns:

$y_1'' + y_1 = 40 \cos \omega t$ $\quad y_{1h} = c_1 \cos t + c_2 \sin t$ $\quad y_{1p} = c_5 \cos \omega t + c_6 \sin \omega t$

$y_2'' + 4y_2 = 20 \cos \omega t$ $\quad y_{2h} = c_3 \cos 2t + c_4 \sin 2t$ $\quad y_{2p} = c_7 \cos \omega t + c_8 \sin \omega t$

$y_{1p}'' = -\omega^2(c_5 \cos \omega t + c_6 \sin \omega t)$ $\quad y_{1p}'' + y_{1p} = (1-\omega^2)(c_5 \cos \omega t + c_6 \sin \omega t) = 40 \cos \omega t$

$y_{2p}'' = -\omega^2(c_7 \cos \omega t + c_8 \sin \omega t)$ $\quad y_{2p}'' + 4y_{2p} = (4-\omega^2)(c_7 \cos \omega t + c_8 \sin \omega t) = 20 \cos \omega t$

$(1-\omega^2)c_5 = 40$, $(1-\omega^2)c_6 = 0 \rightarrow c_5 = 40/(1-\omega^2)$, $c_6 = 0$

$(4-\omega^2)c_7 = 20$, $(4-\omega^2)c_8 = 0 \rightarrow c_7 = 20/(4-\omega^2)$, $c_8 = 0$

$y_{1p} = \frac{40}{1-\omega^2} \cos \omega t$ $\quad y_{2p} = \frac{20}{4-\omega^2} \cos \omega t$

$$\vec{x}_p = \begin{bmatrix} x_{1p} \\ x_{2p} \end{bmatrix} = y_{1p}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + y_{2p}\begin{bmatrix} -2 \\ 1 \end{bmatrix} = \frac{40}{1-\omega^2}\cos \omega t \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \frac{20 \cos \omega t}{4-\omega^2}\begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$= 20 \cos \omega t \begin{bmatrix} \frac{2}{1-\omega^2} + \frac{-2}{4-\omega^2} \\ \frac{2}{1-\omega^2} + \frac{1}{4-\omega^2} \end{bmatrix} = \frac{20 \cos \omega t}{(1-\omega^2)(4-\omega^2)}\begin{bmatrix} 8-2\omega^2-2+2\omega^2 \\ 9-2\omega^2+1-\omega^2 \end{bmatrix} = \frac{20 \cos \omega t}{(1-\omega^2)(4-\omega^2)}\begin{bmatrix} 6 \\ 9-3\omega^2 \end{bmatrix}$$

$$= \frac{60 \cos \omega t}{(1-\omega^2)(4-\omega^2)}\begin{bmatrix} 2 \\ 3-\omega^2 \end{bmatrix}$$

check, set $\omega = 3$: $\frac{60}{(-8)(-5)}\begin{bmatrix} 2 \\ -6 \end{bmatrix} = \frac{3}{2}\begin{bmatrix} 2 \\ -6 \end{bmatrix} = \begin{bmatrix} 3 \\ -9 \end{bmatrix}$ ✓

plot x_{1p}, x_{2p} versus $\omega = 0..4$.

see vertical asymptotes at natural frequencies $\omega = 1, 2$ near which you get a big response.