HW 7.3.11

$$\begin{bmatrix}x_1'\\x_2'\end{bmatrix} = \underbrace{\begin{bmatrix}1&-2\\2&1\end{bmatrix}}_{A}\begin{bmatrix}x_1\\x_2\end{bmatrix} \qquad \begin{bmatrix}x_1(0)\\x_2(0)\end{bmatrix}=\begin{bmatrix}0\\4\end{bmatrix}$$

$$A \longrightarrow \lambda = \begin{bmatrix}1+2i\\1-2i\end{bmatrix} \quad B=\begin{bmatrix}i&-i\\1&1\end{bmatrix} \rightarrow \vec{x}=B\vec{y}$$

$\downarrow$

$$\begin{bmatrix}y_1'\\y_2'\end{bmatrix} = \begin{bmatrix}1+2i&0\\0&1-2i\end{bmatrix}\begin{bmatrix}y_1\\y_2\end{bmatrix} \rightarrow \begin{matrix}y_1'=(1+2i)y_1\\ y_2'=(1-2i)y_2\end{matrix} \qquad \begin{matrix}y_1=c_1e^{(1+2i)t}\\ y_2=c_2e^{(1-2i)t}\end{matrix}$$

$$\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}i&-i\\1&1\end{bmatrix}\begin{bmatrix}c_1e^{(1+2i)t}\\c_2e^{(1-2i)t}\end{bmatrix} = c_1\left(e^{(1+2i)t}\begin{bmatrix}i\\1\end{bmatrix}\right) + c_2e^{(1-2i)t}\begin{bmatrix}-i\\1\end{bmatrix}$$

$$e^t(\cos 2t + i\sin 2t)\begin{bmatrix}i\\1\end{bmatrix} = e^t\begin{bmatrix}-\sin 2t + i\cos 2t\\ \cos 2t + i\sin 2t\end{bmatrix} = e^t\begin{bmatrix}-\sin 2t\\ \cos 2t\end{bmatrix} + i\,e^t\begin{bmatrix}\cos 2t\\ \sin 2t\end{bmatrix}$$

real soln:

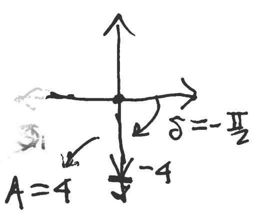
$$\begin{bmatrix}x_1\\x_2\end{bmatrix} = c_1e^t\begin{bmatrix}-\sin 2t\\ \cos 2t\end{bmatrix} + c_2e^t\begin{bmatrix}\cos 2t\\ \sin 2t\end{bmatrix}$$

$$\begin{bmatrix}x_1(0)\\x_2(0)\end{bmatrix} = c_1\begin{bmatrix}0\\1\end{bmatrix} + c_2\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}c_2\\c_1\end{bmatrix} = \begin{bmatrix}0\\4\end{bmatrix} \rightarrow \begin{matrix}c_2=0\\ c_1=4\end{matrix}$$

$$\begin{bmatrix}x_1\\x_2\end{bmatrix} = 4e^t\begin{bmatrix}-\sin 2t\\ \cos 2t\end{bmatrix} \qquad \boxed{x_1=-4e^t\sin 2t,\ x_2=4e^t\cos 2t}$$

$$=4e^t\cos(2t+\tfrac{\pi}{2})$$

shifts to left on t axis so x_1 peaks ahead of $\cos 2t \sim x_2$



NOTE: $e^{2it}\begin{bmatrix}i\\1\end{bmatrix} = e^{2it}\begin{bmatrix}e^{i\pi/2}\\1\end{bmatrix} = \begin{bmatrix}1e^{i(2t+\pi/2)}\\1e^{i(2t)}\end{bmatrix}$ ← x_1 is shifted 90° left, earlier in time compared to x_2 but both have same initial amplitude