

The remaining problems in this section deal with free damped motion. In Problems 15 through 21, a mass m is attached to both a spring (with given spring constant k) and a dashpot (with given damping constant c). The mass is set in motion with initial position x_0 and initial velocity v_0 . Find the position function $x(t)$ and determine whether the motion is overdamped, critically damped, or underdamped. If it is underdamped, write the position function in the form $x(t) = C_1 e^{-pt} \cos(\omega_1 t - \alpha_1)$. Also, find the undamped position function $u(t) = C_0 \cos(\omega_0 t - \alpha_0)$ that would result if the mass on the spring were set in motion with the same initial position and velocity, but with the dashpot disconnected (so $c = 0$). Finally, construct a figure that illustrates the effect of damping by comparing the graphs of $x(t)$ and $u(t)$.

$$mx'' + cx' + kx = 0$$

17. $m = 1, c = 8, k = 16; x_0 = 5, v_0 = -10$