

driven second order DE system: in class exercise

a) $x_1'' = -\frac{(k_1+k_2)}{m_1}x_1 + \frac{k_2}{m_1}x_2 + F_1, x_2'' = \frac{k_2}{m_2}x_1 - \frac{(k_2+k_3)}{m_2}x_2 + F_2, x_1(0)=x_{10}, x_2(0)=x_{20}, x_1'(0)=v_{10}, x_2'(0)=v_{20}$

Rewrite these coupled scalar DEs in matrix form: $\vec{x}''' = A\vec{x} + \vec{F}, \vec{x}'(0) = \vec{x}_0, \vec{x}'(0) = \vec{v}_0$

b) Substitute in the parameter values: $(k_1, k_2, k_3) = (1, 3/2, 1), (m_1, m_2) = (1, 1), x_{10} = 1, x_{20} = 0, v_{10} = 0, v_{20} = 1$
 $F_1 = 0, F_2 = 50 \cos 3t \leftarrow t \text{ is independent variable}$

c) Use Maple to find the integer eigenvalues $\lambda_1 > \lambda_2$ (ordered by increasing absolute value)
eigenvectors \vec{b}_1, \vec{b}_2 . Let $B = \langle \vec{b}_1 | \vec{b}_2 \rangle, \vec{x} = B\vec{y}, \vec{y} = B^{-1}\vec{x}$.

d) For the transformed DEs: $B^{-1}[(B\vec{y})'' = A(B\vec{y}) + \vec{F}] \rightarrow \vec{y}'' = \underbrace{B^{-1}AB}_{AB}\vec{y} + \underbrace{B^{-1}\vec{F}}_{\vec{F}_B}$,
confirm that $A_B = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$ by Maple matrix multiplication and evaluate \vec{F}_B .

e) Now write down the scalar DEs for the decoupled variables (y_1, y_2) .

f) Solve each decoupled DE for its general solution using the method of undetermined coefficients.

g) Express $\vec{x} = B\vec{y}$ in terms of these without carrying out the matrix multiplication.

h) Use matrix methods to impose the initial conditions on this general solution for \vec{x}

$$\vec{x}(0) = B\vec{y}(0), \vec{x}'(0) = B\vec{y}'(0) \longrightarrow \underbrace{\vec{y}(0) = B^{-1}\vec{x}(0)}, \underbrace{\vec{y}'(0) = B^{-1}\vec{x}'(0)}$$

gives 2 arb constants, then solve this for remaining two constants

i) Backsub your values for the 4 arbitrary constants into \vec{x} and express the result in scalar form to identify $x_1(t), x_2(t)$ individually and then express \vec{x} in vector form as a linear combination of eigenvectors.

j) Now enter into Maple the original scalar DEs with part b) values, separated by commas together with the 4 initial conditions. Right click on output and solve. Write down the solution.

k) Compare with your hand solution. They should agree. If not there is an error somewhere.

l) Separate the homogeneous and particular vector solutions $\vec{x} = \vec{x}_h + \vec{x}_p$ that arise from those terms in $\vec{y} = \vec{y}_h + \vec{y}_p$. Express $\vec{x}_p = y_3(t)\vec{b}_3$, where \vec{b}_3 is the lowest integer constant vector proportional to \vec{x}_p . Write out the final vector form of the solution

$$\vec{x} = \underbrace{y_{1h}\vec{b}_1 + y_{2h}\vec{b}_2}_{\text{natural modes of system}} + \underbrace{y_3\vec{b}_3}_{\text{response mode of system}}$$

m) Re-express the sinusoidal functions y_{1h}, y_{2h}, y_3 in phase-shifted cosine form in this vector form of the solution, identifying their amplitudes A_1, A_2, A_3 .

Make a hand plot of the 6 vectors: $\pm A_1 \vec{b}_1, \pm A_2 \vec{b}_2, \pm A_3 \vec{b}_3$.

determine parallelogram

$$|y_{1h}| \leq A_1, |y_{2h}| \leq A_2$$

shade this in

