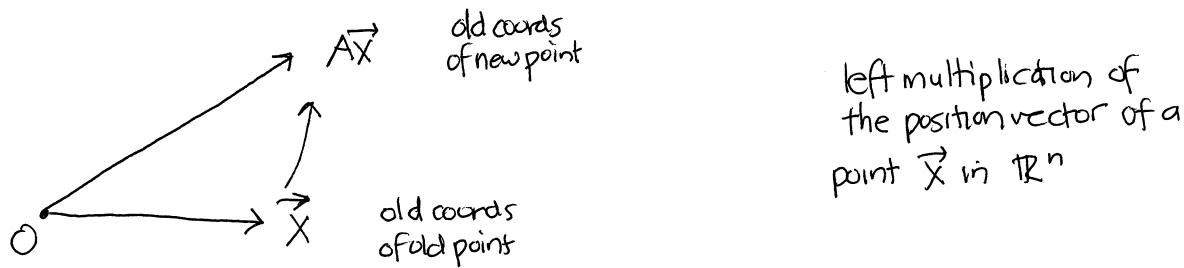


"Diagonalization" of a square matrix by a basis of eigenvectors



left multiplication of
the position vector of a
point \vec{X} in \mathbb{R}^n

change of coordinates

$$\vec{x} = y_1 \vec{b}_1 + \dots + y_n \vec{b}_n = \langle \vec{b}_1 | \dots | \vec{b}_n \rangle \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = B \vec{y} \quad \text{so} \quad \vec{x} = B \vec{y}$$

Summary:
$$\boxed{\begin{array}{ll} \vec{x} = B \vec{y} & , \\ \text{old} & \text{new} \\ \vec{y} = B^{-1} \vec{x} & , \\ \text{new} & \text{old} \end{array}}$$

$$B^{-1} \vec{x} = B^{-1} B \vec{y} = \vec{y}$$

re-express $\vec{x} \rightarrow A\vec{x}$ in new coords

new coords of old point: \vec{y}

B ↴ old coords of old point: $B \vec{y} = \vec{x}$

A ↴ old coords of new point: $A B \vec{y} = A \vec{x}$

B^{-1} ↴ new coords of new point: $B^{-1} A B \vec{y} = B^{-1} A \vec{x}$ $(B^{-1} A B) \vec{y}$



Conclusion: if old coords undergo $\vec{x} \rightarrow A\vec{x}$,
then new coords undergo $\vec{y} \rightarrow \underline{(B^{-1} A B)} \vec{y}$

$A_B \equiv B^{-1} A B$ represents the action
of left multiplication by A in the
new coord system

eigenbasis

If $\{\vec{b}_i\}$ is a basis of eigenvectors of A : $A \vec{b}_i = \lambda_i \vec{b}_i$

$$\text{then } A(y_1 \vec{b}_1 + \dots + y_n \vec{b}_n) = y_1 A \vec{b}_1 + \dots + y_n A \vec{b}_n = y_1 (\lambda_1 \vec{b}_1) + \dots + y_n (\lambda_n \vec{b}_n) = (\lambda_1 y_1) \vec{b}_1 + \dots + (\lambda_n y_n) \vec{b}_n$$

$$\text{so } \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \rightarrow \begin{bmatrix} \lambda_1 y_1 \\ \vdots \\ \lambda_n y_n \end{bmatrix} = \underbrace{\begin{bmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \ddots & \ddots & 0 \\ \vdots & & \ddots & \lambda_n \end{bmatrix}}_{B^{-1} A B} \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$$

$B^{-1} A B$ is a diagonal matrix whose diagonal entries are the eigenvalues corresponding to the eigenvectors in the same order

The matrix which represents left multiplication by A in the old coords
is represented by left multiplication by a diagonal matrix $A_B = B^{-1} A B$ in the new coords.
The matrix is said to be "diagonalized" by its basis of eigenvectors.