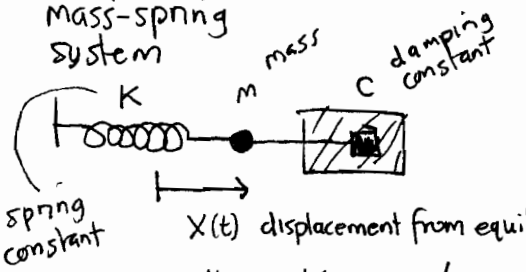
# damped harmonic oscillator systems in nature

$$Ay'' + By' + Cy = 0 \rightarrow \quad y'' + k_0 y' + \omega_0^2 y = 0$$

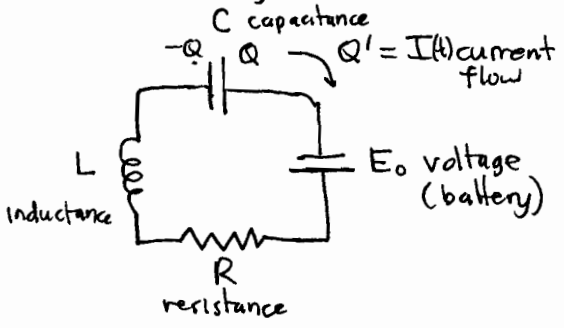
A, B, C all same sign

- damped mass-spring system



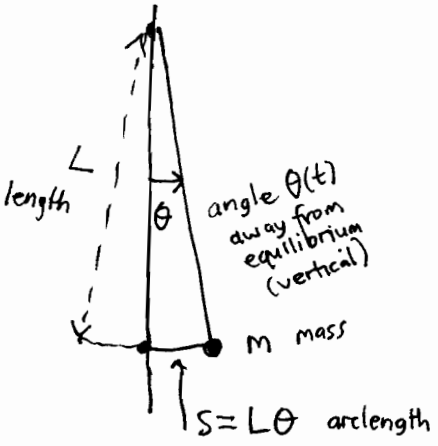

$$mx'' = -Kx - cx' \rightarrow \quad mx'' + cx' + Kx = 0 \rightarrow x'' + \frac{c}{m}x' + \frac{K}{m}x = 0$$

- RLC circuit (details later) with battery




$$LI'' + RI' + \frac{1}{C}I = 0 \rightarrow I'' + \frac{R}{L}I' + \frac{1}{LC}I = 0$$

- small oscillations of a pendulum with "air resistance"




$$mS'' = \text{acceleration} = mL\theta''$$

$$mL\theta'' = -mg\theta - cL\theta'$$

approx g-force (tangential); damping ∝ velocity $L\theta'$

$$mL\theta'' + cL\theta' + mg\theta = 0 \rightarrow \theta'' + \frac{c}{m}\theta' + \frac{g}{L}\theta = 0$$

direct analogy between corresponding coefficients

one easily identifies natural frequency $\omega_0$ and decay rate $k_0$ in each system

solve once and for all in terms of $y, k_0, \omega_0$ and then identify values for individual systems