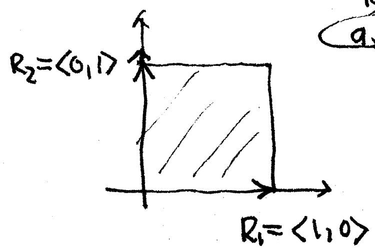
2705 only needs to know is $|A|$ zero or nonzero, in which case the square matrix A is singular (no inverse) or nonsingular (has an inverse).

However the absolute value of the determinant is a measure of area (2×2), volume (3×3), etc, as already seen in Calc 3 with areas of parallelograms and volumes of parallelopipeds. Here's why:

$\boxed{\text{determinants and area}}$ etc

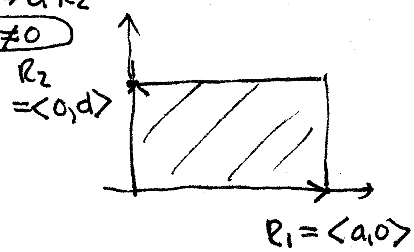
we interpret the rows of a 2×2 matrix as vectors in $\mathbb{R}^2$.

$$I_2=\begin{bmatrix}1&0\\0&1\end{bmatrix}\xrightarrow[\substack{R_1\to aR_1\\ R_2\to dR_2\\ a,d\neq 0}]{\text{(scaleupstep)}}\begin{bmatrix}a&0\\0&d\end{bmatrix}\xrightarrow[R_1\to R_1+kR_2]{}\begin{bmatrix}a&kd\\0&d\end{bmatrix}$$



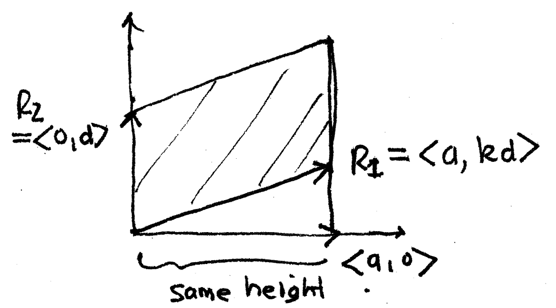
det $=1\cdot 1=1$

Area $=1$



det $=ad$

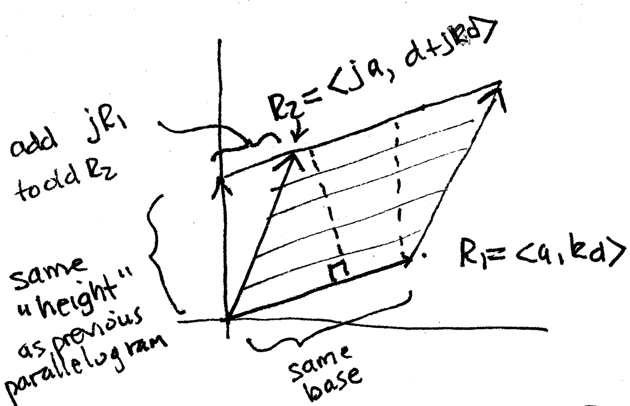
Area $=|ad|$



det $=ad$

Area $=$ (base)(height) $=|ad|$

$$\xrightarrow[R_2\to R_2+jR_1]{}\begin{bmatrix}a&kd\\ja&d+jkd\end{bmatrix}$$



det $=ad$ $\left[=a(d+jkd)-ja(kd)\right]$

Area $=|ad|$

$=\left|\det\begin{pmatrix}a&kd\\ja&d+jkd\end{pmatrix}\right|$

The addrow operations create new parallelograms with the same area as the preceding ones. Together with swaprows, the area is preserved. Only the multiply row operations (by nonzero numbers!) change the area.

Gaussian reduction without the multiply row operations basically goes backwards taking a general matrix back to one aligned with the axes where area is just ~~the~~ the product of side lengths.

This generalizes to n vectors forming a parallelopiped in $\mathbb{R}^n$ ($\mathbb{R}^3$ is familiar from Calc3). The n vectors are rows in a square matrix & the abs value of the determinant is the n-dimensional volume of the parallelopiped they form.

If the vectors are not all "independent" the volume is zero (2 collinear vectors in $\mathbb{R}^2$, 3 coplanar or even collinear vectors in $\mathbb{R}^3$)

Note: diagrams are drawn as if a,d are positive, but this holds for any sign.