$\boxed{\text{post 7.3}}$

so far:

$\vec{x}' = A\vec{x}$ $\quad A = \begin{bmatrix}0&1\\-2&-3\end{bmatrix} \to$ $\begin{array}{c} \lambda = \;\; -1 \quad -2 \\ B_{maple} = \begin{bmatrix}-1&-1/2\\1&1\end{bmatrix} \\ \times 1 \quad \times -2 \end{array} \to$ $\begin{array}{c}\lambda = -1 \;\; -2 \\ B = \begin{bmatrix}1&1\\-1&-2\end{bmatrix}\end{array}$

$A_B = \begin{bmatrix}-1&0\\0&-2\end{bmatrix}$ $\qquad B^{-1} = \begin{bmatrix}2&1\\-1&-1\end{bmatrix}$

we are free to scale eigenvectors as we choose

we can make first entry 1 instead of last

↓ generalize

$\vec{x}' = A\vec{x} + \vec{F}$ or $\boxed{\vec{x}'' = A\vec{x} + \vec{F}}$ nonhomogeneous term "forcing function" and change to second order derivatives

both are solved in same way as original homogeneous DE system.

$\vec{x} = B\vec{y}$, $\vec{y} = B^{-1}\vec{x}$ $\qquad$ we do second order case as an example:

$$B^{-1}\left[(B\vec{y})'' = A(B\vec{y}) + \vec{F}\right]$$

$$\vec{y}'' = \underbrace{(B^{-1}AB)}_{A_B}\vec{y} + \underbrace{B^{-1}\vec{F}}_{\text{new components of } \vec{F}} \qquad B^{-1}\vec{F} = \begin{bmatrix}2&1\\-1&-1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}3\\-2\end{bmatrix}$$

$$\begin{bmatrix}y_1''\\y_2''\end{bmatrix} = \begin{bmatrix}-1&0\\0&-2\end{bmatrix}\begin{bmatrix}y_1\\y_2\end{bmatrix} + \begin{bmatrix}3\\-2\end{bmatrix} = \begin{bmatrix}-y_1+3\\-2y_2-2\end{bmatrix}$$

backsub into DE:

$y_1'' = -y_1 + 3$ $\quad y_1'' + y_1 = 3$ $\quad y_{1h} = c_1\cos t + c_2 \sin t$ $\quad y_{1p} = C_5$ $\quad 0 + C_5 = 3$ $\quad C_5 = 3$

$y_2'' = -2y_2 - 2$ $\quad y_2'' + 2y_2 = -2$ $\quad y_{2h} = C_3 \cos\sqrt{2}t + C_4 \sin\sqrt{2}t$ $\quad y_{2p} = C_6$ $\quad 0 + 2C_6 = -2$ $\quad C_6 = -1$

$y_1 = y_{1h} + y_{1p}$, $y_2 = y_{2h} + y_{2p}$

$$\begin{bmatrix}x_1\\x_2\end{bmatrix} = y_1\vec{b}_1 + y_2\vec{b}_2 = (c_1\cos t + c_2\sin t + 3)\begin{bmatrix}1\\-1\end{bmatrix} + (c_3\cos\sqrt{2}t + c_4\sin\sqrt{2}t - 1)\begin{bmatrix}1\\-2\end{bmatrix}$$

$$= \underbrace{\underbrace{(c_1\cos t + c_2\sin t)\begin{bmatrix}1\\-1\end{bmatrix}}_{\omega_1 = 1 \text{ slow mode}} + \underbrace{(c_3\cos\sqrt{2}t + c_4\sin\sqrt{2}t)\begin{bmatrix}1\\-2\end{bmatrix}}_{\omega_2 = \sqrt{2} \text{ fast mode}}}_{\vec{x}_h} + \underbrace{3\begin{bmatrix}1\\-1\end{bmatrix} - \begin{bmatrix}1\\-2\end{bmatrix}}_{\begin{bmatrix}2\\-1\end{bmatrix}}$$

(agrees with Maple coefficients c_1, c_2, c_3, c_4)

$\vec{x}_h$ natural behavior of system without applied force or "driving term" $\vec{F}$

$\vec{x}_p$ = response mode as a result of $\vec{F}$ being present.

$\boxed{\text{initial conditions}}$ set 4 arbitrary constants: $x_1(0) = 1$, $x_2(0) = 0$, $x_1'(0) = 0$, $x_2'(0) = 1$

$$\begin{bmatrix}x_1'\\x_2'\end{bmatrix} = \begin{bmatrix}1&1\\-1&-2\end{bmatrix}\begin{bmatrix}-c_1\sin t + c_2\cos t\\-\sqrt{2}c_3\sin\sqrt{2}t + \sqrt{2}c_4\cos\sqrt{2}t\end{bmatrix} + \begin{bmatrix}0\\0\end{bmatrix}$$

$$\begin{bmatrix}x_1'(0)\\x_2'(0)\end{bmatrix} = \begin{bmatrix}1&1\\-1&-2\end{bmatrix}\begin{bmatrix}c_2\\\sqrt{2}c_4\end{bmatrix} = \begin{bmatrix}0\\1\end{bmatrix} \to \begin{bmatrix}c_2\\\sqrt{2}c_4\end{bmatrix} = \begin{bmatrix}2&1\\-1&-1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}1\\-1\end{bmatrix} \to \begin{bmatrix}c_2\\c_4\end{bmatrix} = \begin{bmatrix}1\\-1/\sqrt{2}\end{bmatrix}$$

$$\begin{bmatrix}x_1(0)\\x_2(0)\end{bmatrix} = \begin{bmatrix}1&1\\-1&-2\end{bmatrix}\begin{bmatrix}c_1\\c_3\end{bmatrix} + \begin{bmatrix}2\\-1\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix} \to \begin{bmatrix}c_1\\c_3\end{bmatrix} = \begin{bmatrix}2&1\\-1&-1\end{bmatrix}\begin{bmatrix}1-2\\0+1\end{bmatrix} = \begin{bmatrix}-1\\0\end{bmatrix}$$

$$\begin{bmatrix}x_1\\x_2\end{bmatrix} = (-\cos t + \sin t)\begin{bmatrix}1\\-1\end{bmatrix} + \left(-\tfrac{1}{\sqrt{2}}\sin\sqrt{2}t\right)\begin{bmatrix}1\\-2\end{bmatrix} + \begin{bmatrix}2\\-1\end{bmatrix}$$ vector form

$$= \begin{bmatrix}2 - \cos t + \sin t - \tfrac{1}{\sqrt{2}}\sin\sqrt{2}t\\-1 + \cos t - \sin t + \sqrt{2}\sin\sqrt{2}t\end{bmatrix}$$ scalar form